Polynomiography as a Visual Tool: Building Meaning from Images

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Overview

- Research-based Perspective
- Centrality of Representations
- Graphical Images as Representations
- An Example in Secondary Mathematics
- Observations and Future Directions

Research on Mathematical Learning

- Longitudinal and Cross Sectional Studies (Maher, 2009)
- Over Two Decades of Research (Maher, 2002, 2005, 2009)
- Underestimate What Students Can Do (Maher & Martino, 1996, 1999; Maher, Powell, & Uptegrove (Eds.), in press)

Findings

- At a very early age, students can build the idea of mathematical proof
- Early representations are key
- Early ideas become elaborated and later re-represented in symbolic forms

Research Approach

- Videotape students engaged in exploring rich tasks (Francisco & Maher, 2005)
- Analyze videos to flag for critical ideas (Powell, Francisco & Maher, 2004, 2003)
- Follow up with student interviews
- Build new tasks to invite further exploration

A Recent Exploration

- High School Students Studying Polynomials
- A Motivated Teacher (Kevin)
- An Example: The Case of Ankita

Seven polynomials in 50 minutes

1) $y = x^2 - 4$

2) $y = x^3 - 3x^2 - x + 3$

3) $y = x^2 + 4$

4) $y = x^4 - 16$

Seven polynomials in 50 minutes

5) $y = x^3 - 1$

6) $y = x^3 + 1$

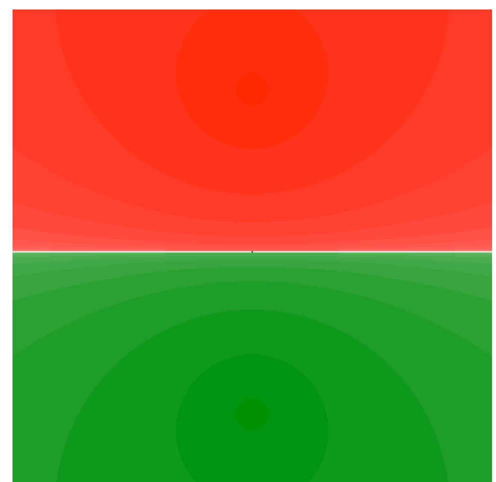
7) $y = x^4 + 13x^2 + 36$

After 16 minutes

Ankita points at the images for #1 and #3 and says they look similar, like, tilted on the side, not opposites, but similar

1) $y = x^2 - 4$

3) $y = x^2 + 4$



Written work for polynomials 1 and 3

Symbolic representation by student working on solutions for polynomials

1) $y = x^2 - 4$

xaxis = ±2
yaxis = −2

1) $0 = x^2 - 4$

$\boxed{x = \pm 2}$

3) $y = x^2 + 4$

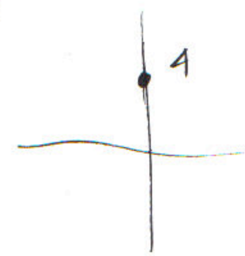
3) $y = x^2 + 4$

$0 = x^2 + 4$

$-4 = x^2$

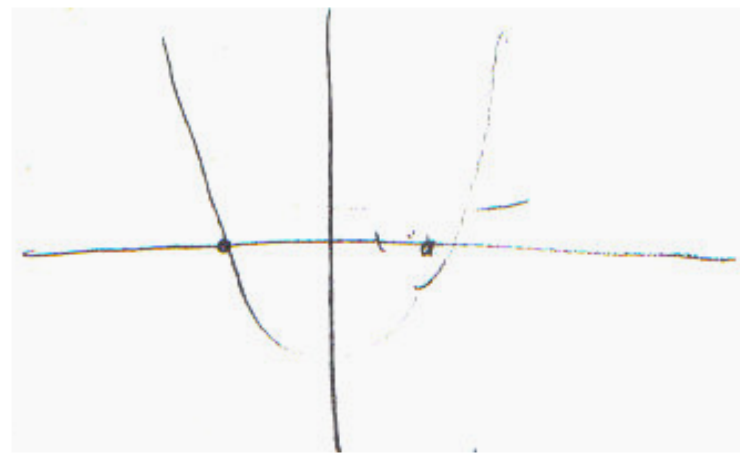
$x = \pm\sqrt{4}$

$\pm 2i$



Graphical work for polynomial 1

Ankita's traditional graphic representation



After 17 minutes

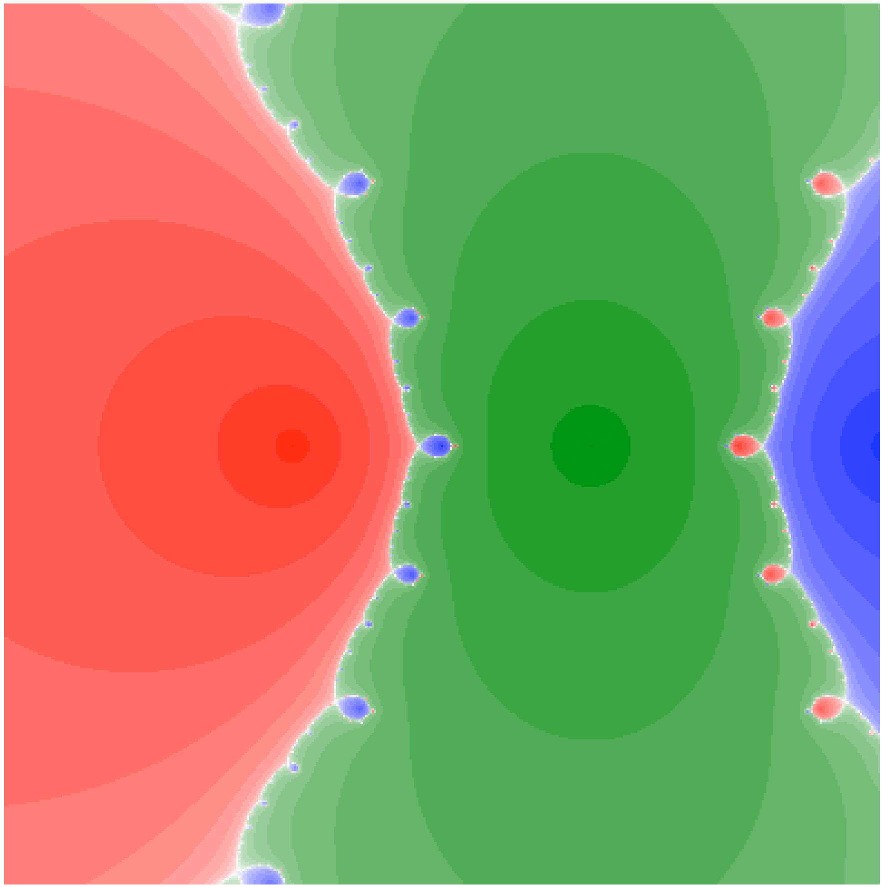
Ankita speculates a relationship between the degree of the equation and the number of colors in each image after seeing graphs for equations 2) and 4)

2) $y = x^3 - 3x^2 - x + 3$

4) $y = x^4 - 16$

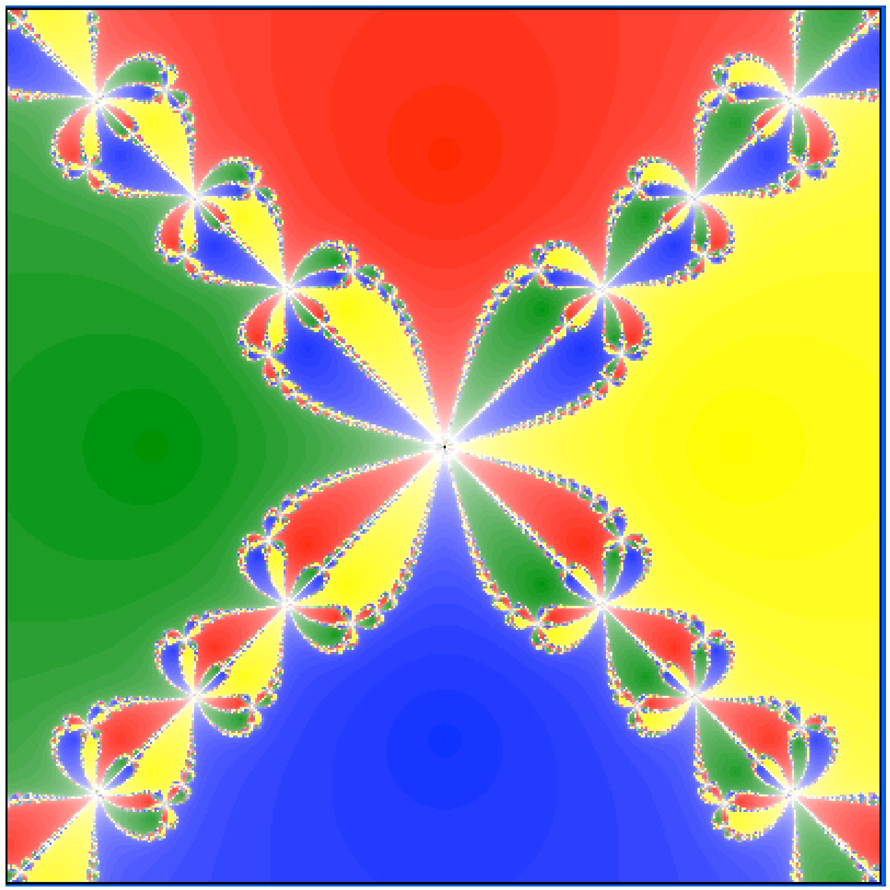
After 17 minutes

2) $y = x^3 - 3x^2 - x + 3$



After 17 minutes

4) $\quad y = x^4 - 16$



Written work for polynomials 2)

Ankita's written work with traditional, graphic image

$$2) \quad y = x^3 - 3x^2 - x + 3$$

$2)\ y = x^3 - 3x^2 - x + 3$

$0 = x^3 - 3x^2 + x + 3$

$-3 = x(x^2 - 3x + 1)$

y axis: 3

x axis

Written work for polynomials 4)

Ankita's written work with points plotted for sketching graphic image

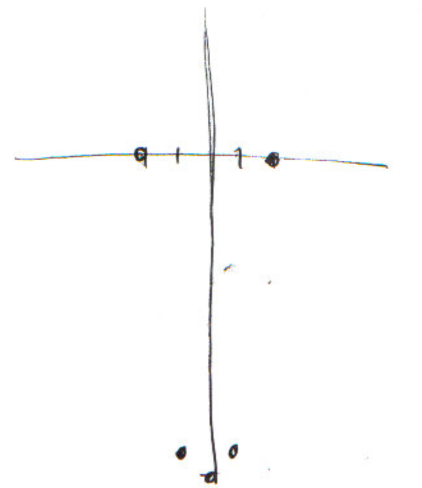
$$4) \quad y = x^4 - 16$$

4) $y = x^4 - 16$

$y = (x^2+4)(x^2-4)$

$x = \pm 2$

$y = -16$

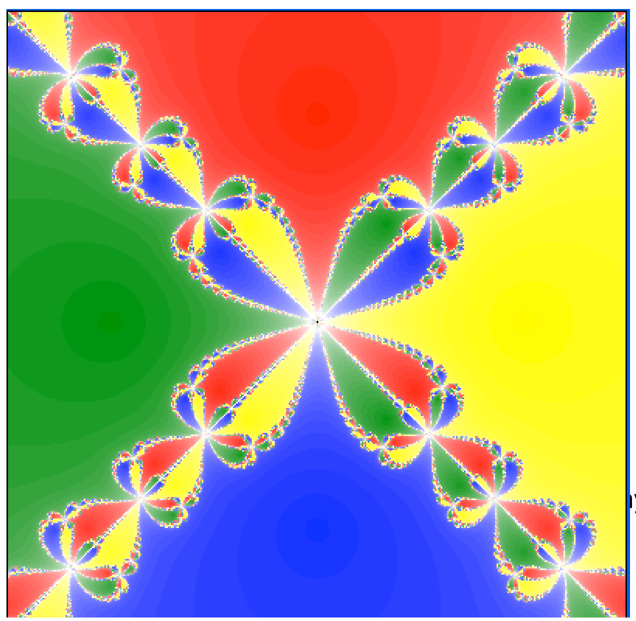


After 28 minutes

Ankita's rewrite of the fourth-degree polynomial as the product of its factors

$$(x^2 - 4)(x^2 + 4) = x^4 - 16$$

After 28 minutes



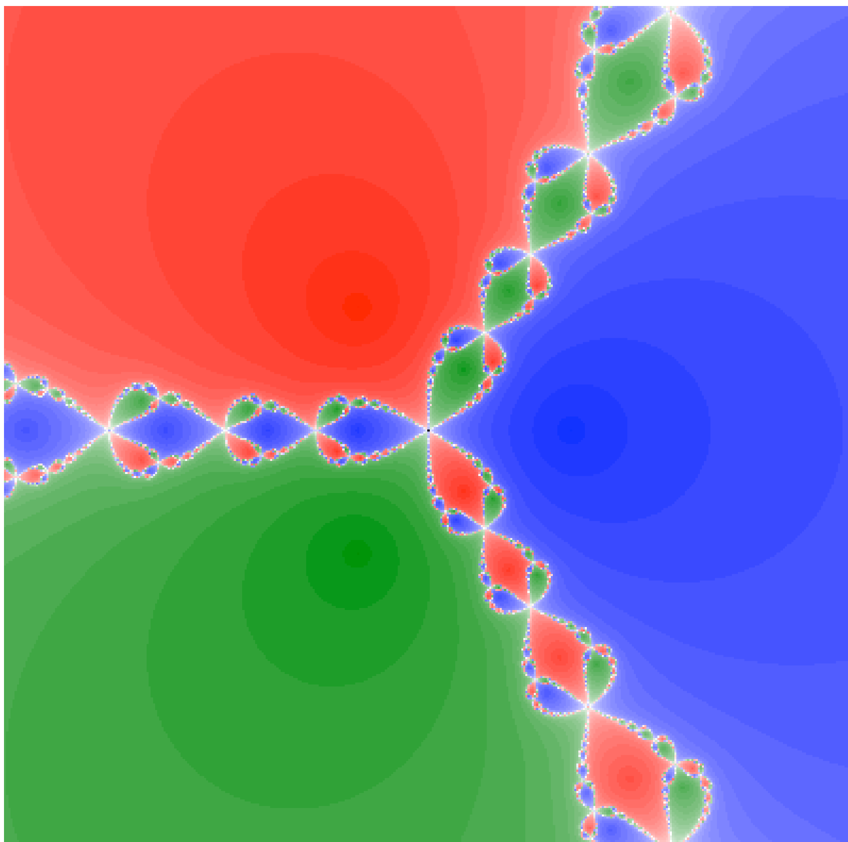
After 30 minutes

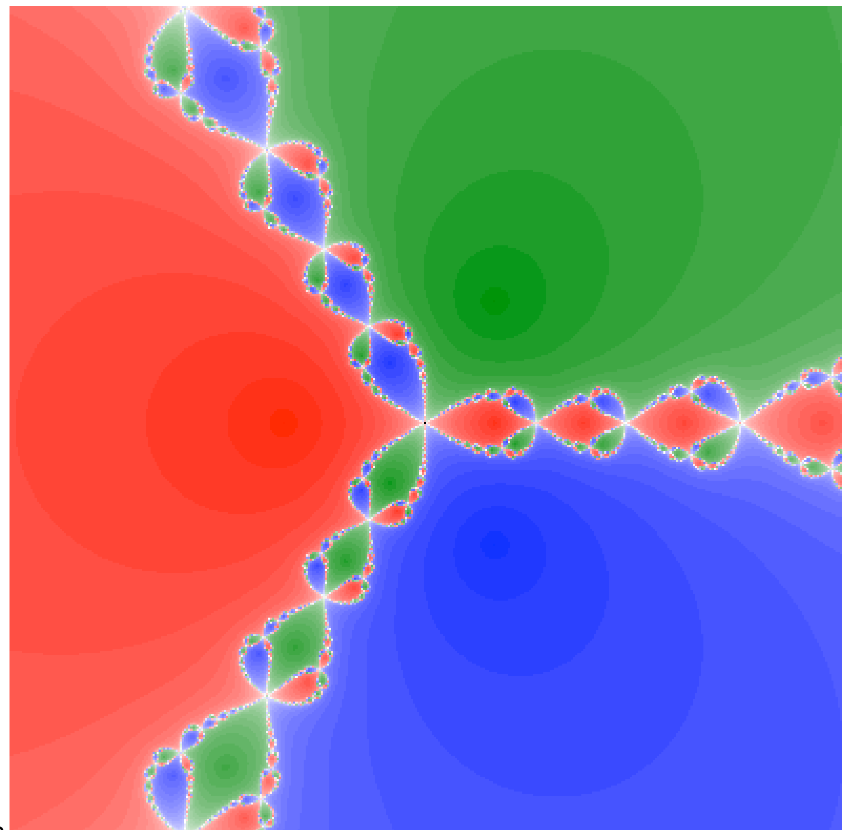
Ankita's observation of a possible rotational relationship between equations 5) and 6)

5) $y = x^3 - 1$ 6) $y = x^3 + 1$

After 30 minutes

5) $y = x^3 - 1$ 6) $y = x^3 + 1$

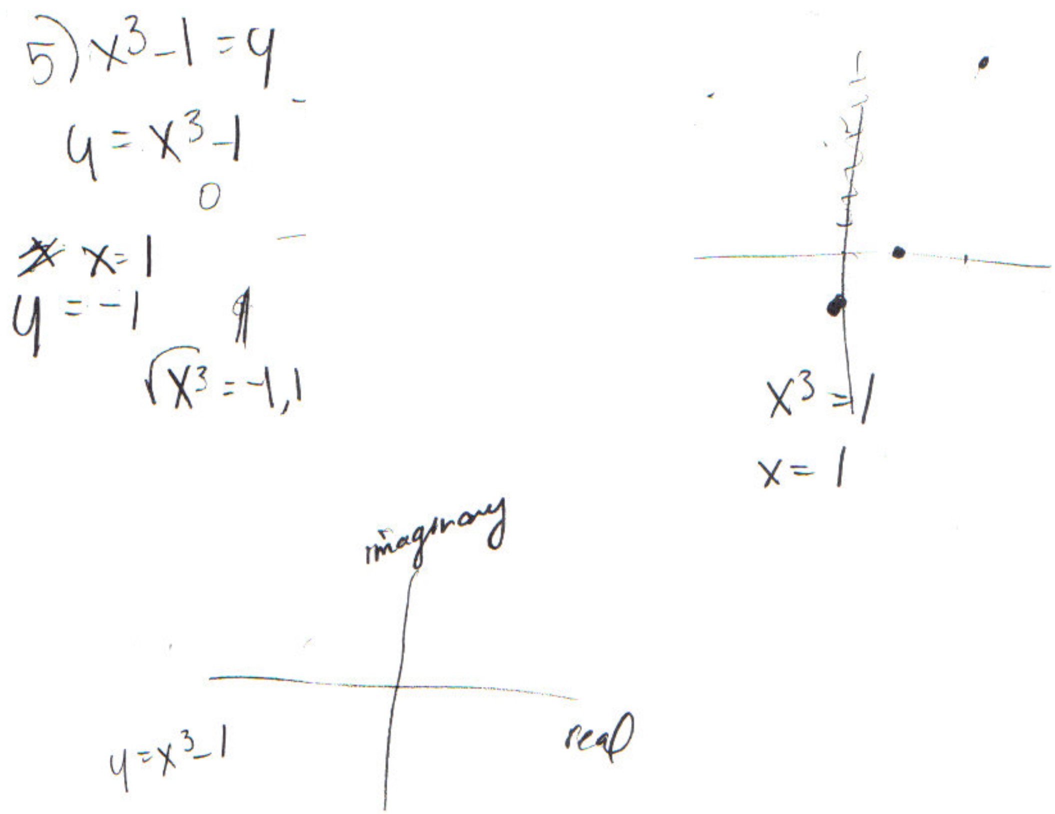




Written work for polynomial 5)

$$5)\quad y = x^3 - 1$$

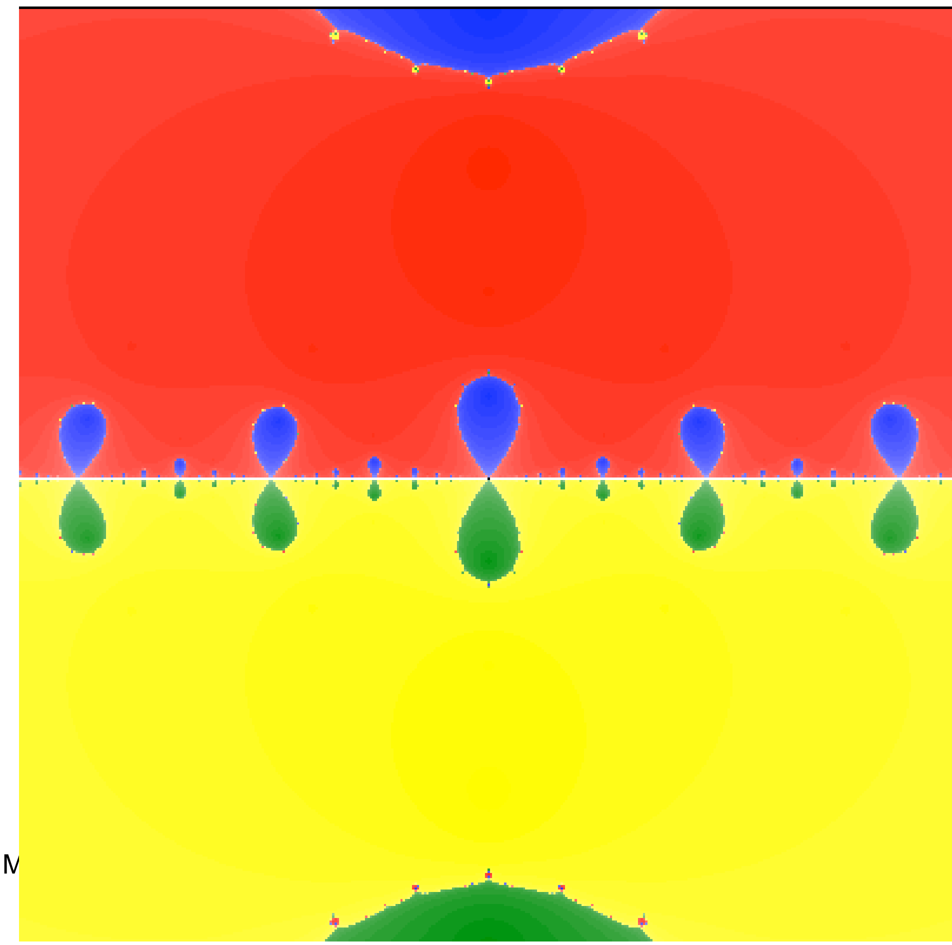
Ankita's written work with points plotted on axes



After 36 minutes

Ankita wondering about the relationship of each color's dark regions to polynomial solutions using the complex plane

7) $y = x^4 + 13x^2 + 36$



Written work for polynomial 7)

Ankita's written work for polynomial 7)

$$7) \quad y = x^4 + 13x^2 + 36$$

$y = x^4 + 13x^2 + 36$

7) $0 = x^4 + 13x^2 + 36$

$(x^2+9)(x^2+4)$

$x^2+9=0$

$x^2=-9$

x^4+13x^2+36

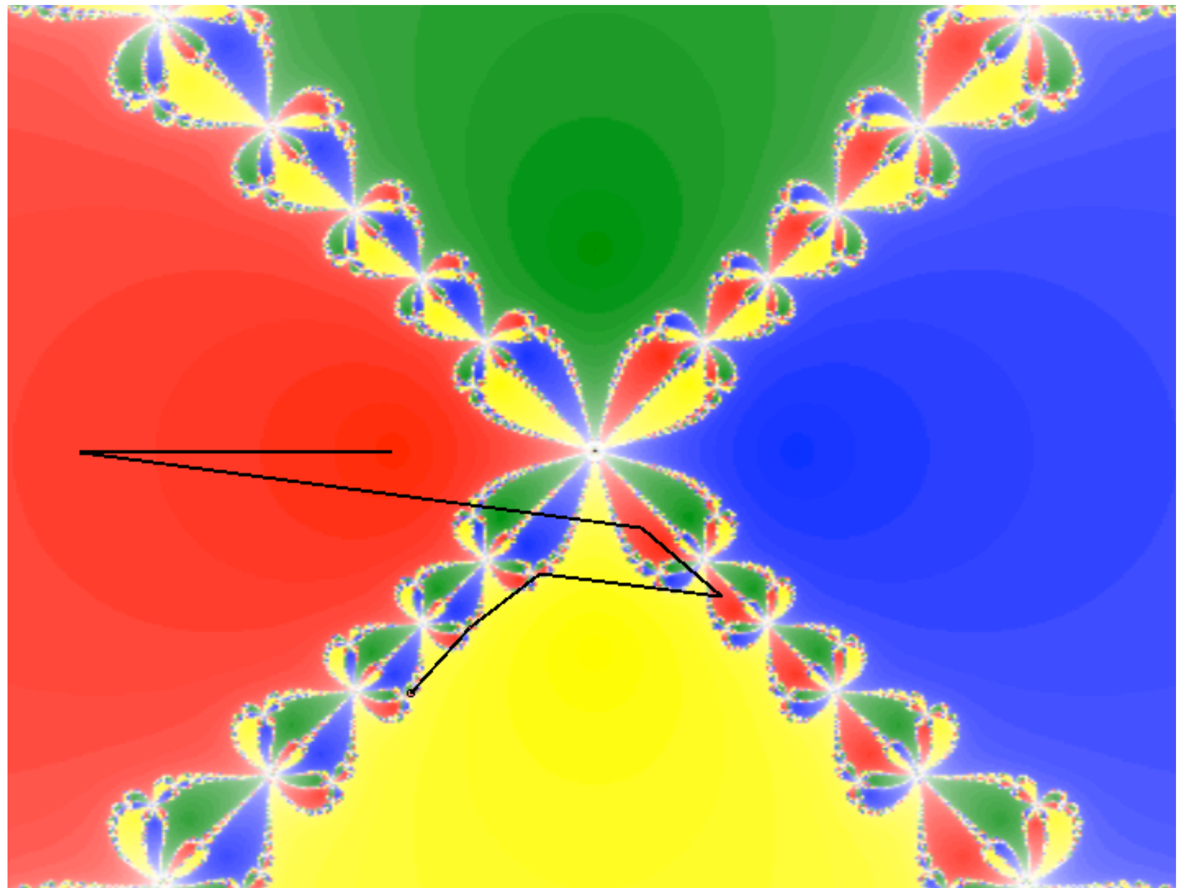
$(x^2+9)(x^2+4)$

$(0,36)=y$

$\pm 39 \quad \pm 29$

After 43 minutes

Ankita's conjecture that the path might be the result of multiple iterations, represented by a point in the complex plane



45-49 minutes

Ankita confirms idea of iterations, saying:

"you follow the answer, you keep plugging it back in"

- (point at screen) "I plug this in and I get this." (following the iteration path)
- "… a smaller one goes to a bigger one.." (the rule of ponds)
- Observes that in every instance the iteration path stayed in the same color

Conclusions / Suggestions

- Students Do Engage with Graphic Representations
- Students Can Connect Graphic and Symbolic Representations

Questions for Study

- How Might Polynomial/Art Investigations Be Made Accessible to Teachers and Their Students?
- To What Extent Can Investigations of Polynomials, Complex Solutions,Iterations and the Images Generated Contribute to Building Mathematical Ideas?
- To What Extent Are Students Motivated by their Artistic Creativity to Explore in Greater Depth the Underlying Mathematical Ideas?

THANK YOU

and

Thank You, Ankita!