- The Mathematics of Design

- Introduction to a course developed
- By Jay Kappraff at NJIT

Pedagoqical Levels

- 1. Metaphor and creativity
- 2. Two and three dimensional design concepts
- 3. Mathematical concepts – geometry and algebra
- 4. Communications and literacy
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Pedagogical Objectives

- Topics are arranged into independent modules. A spiral model of learning is used rather than a linear model --- Concepts return in different contexts. Most topics are connected at different levels.
- Each module should contain significant mathematical content.
- Course addresses a variety of design ideas such as: symmetry, symmetry breaking, duality, positive and negative space, mathematical constraints on space, the nature of infinity, modular design, etc.

Pedagogical Objectives (continued)

- Algorithms to carry out design activities are emphasized rather than basic theory.
- Designs derived from different cultures both ancient and modern are emphasized.
- Most design activities are either adapted to the computer or are computer applicable. However, the first stage of the design process is generally hands on or constructive.
- Materials are ungraded – they can be adapted to students from the 3rd grade to students on the graduate level, both mathematically oriented and non-mathematical students
- The course emphasizes writing and communications.

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Evaluation of Students

- Scrapbooks
- Journals
- Design projects
- Homework exercises
- Essays
- No examinations

Main Topics

- Informal Geometry
- Projective Geometry
- Theory of Graphs
- Theory of Proportions
- Fractals
- Modular Tilings
- Three-Dimensional Geometry and polyhedra
- Theory of Knots and Surfaces
- Symmetry and Music

Examples of Modules in this Presentation

- 2-D and 3-D lattice designs
- Proportional system of Roman Architecture (silver mean)
- Golden mean and Le Corbusier's Modulor
- Brunes star
- Tangrams and Amish quilts
- Sona and Lunda tilings
- Penrose tilings
- Hyperbolic geometry
- Projective geometry and design
- Lindenmayer L-systems and fractals
- Traveling salesman problem and design
- Application of fractals to image processing
- Spacefilling curves and image compressing
- Music – the diatonic scale and clapping patterns

Informal Geometry

1. Tangrams and Amish quilt patterns
2. Brunes Star
3. Coffee can cover geometry
4. Baravelle spiral

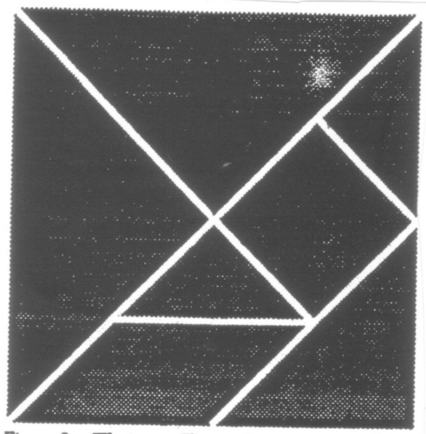


Figure 3a. The seven Tangram pieces formed by dissecting a square.

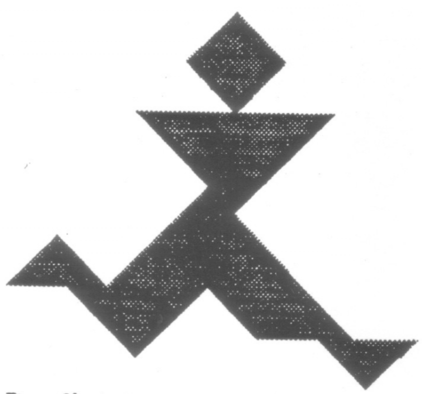
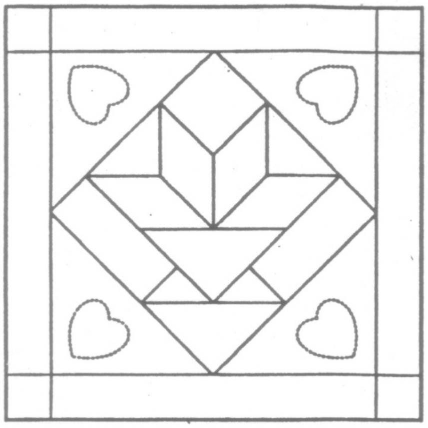
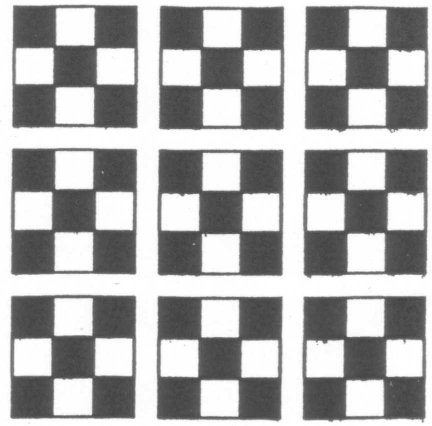
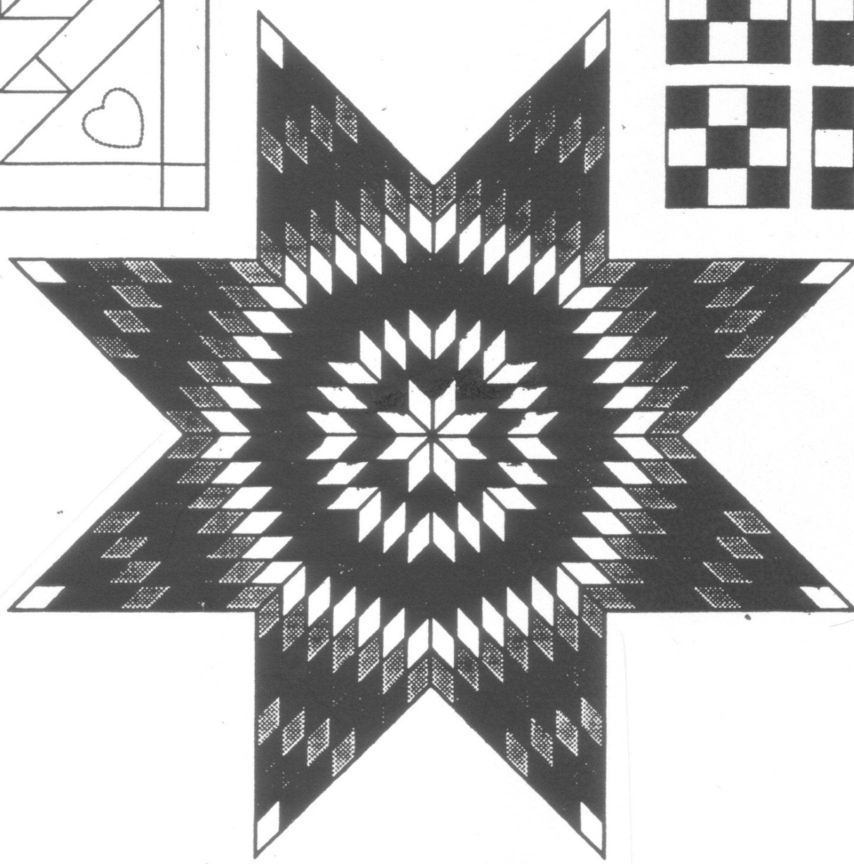


Figure 3b. A pictogram constructible from the Tangram pieces. Try it!

COFFEE CAN COVER GEOMETRY

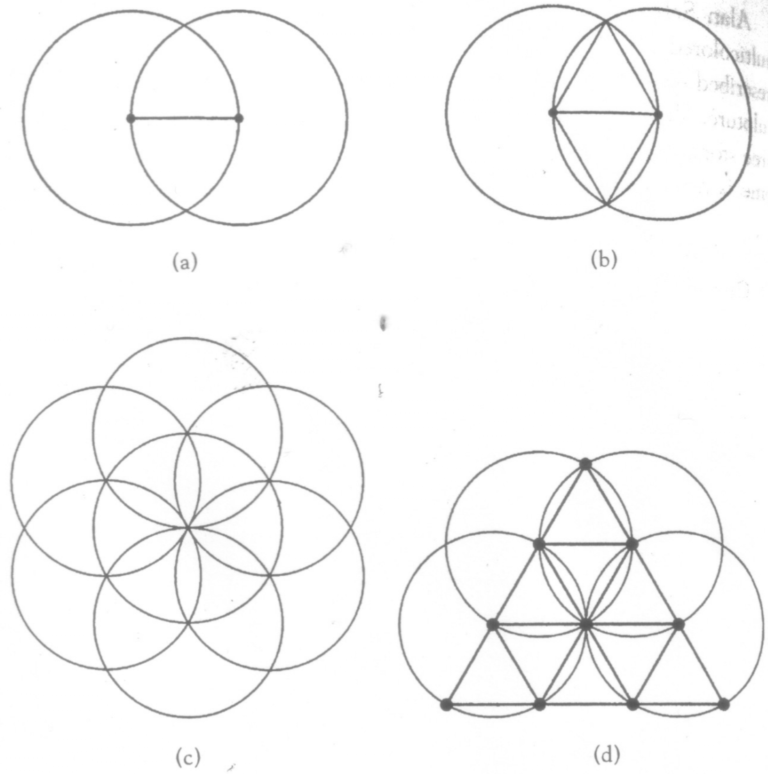


Figure 6.A1 Creation of a triangular grid of circles.

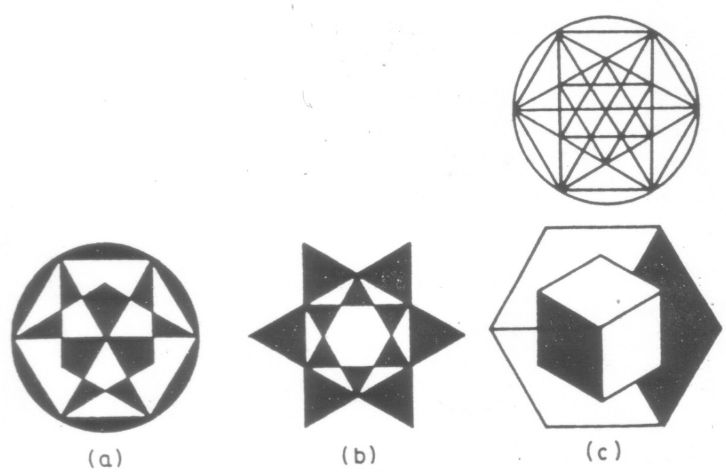
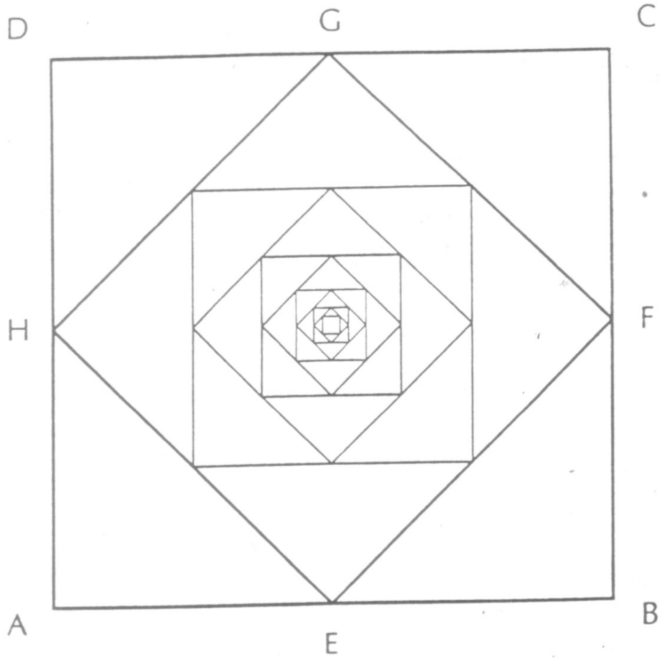


Figure 5.1 The versatility of the triangular grid. These are created by shading a portion of the grid enclosed by the circle.

Baravelle Spiral within a Square



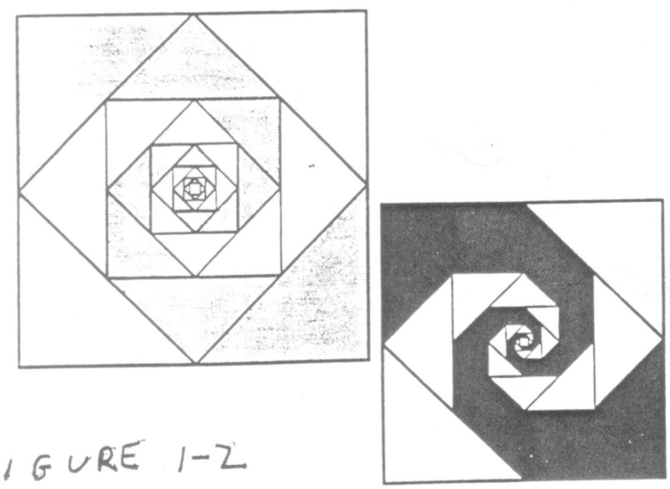


FIGURE 1-2

Given square ABCD

. Locate the midpoints of $\overline{\text{AB}}$, $\overline{\text{BC}}$, $\overline{\text{CD}}$ and $\overline{\text{DA}}$.

. Label them E, F, G and H, espectively.

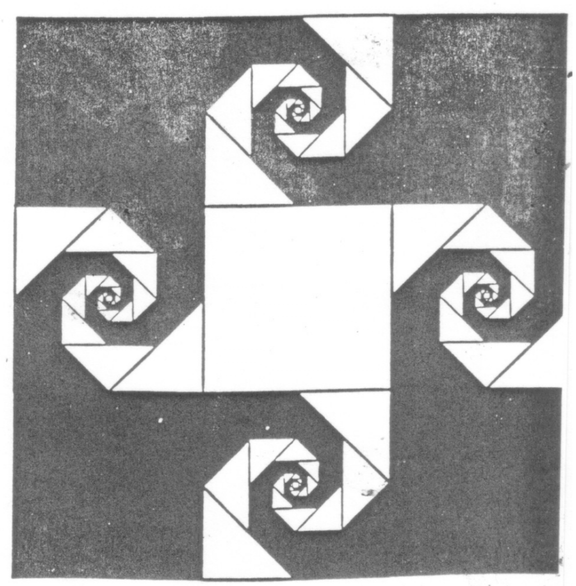
. Join them with line seg- nents to form square EFGH.

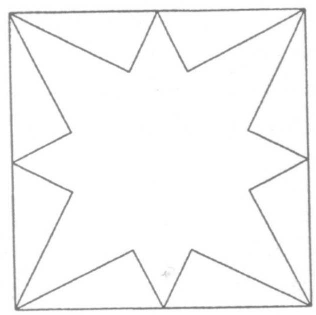
. In the same way, join the nidpoints of the sides of FGH to form another square.

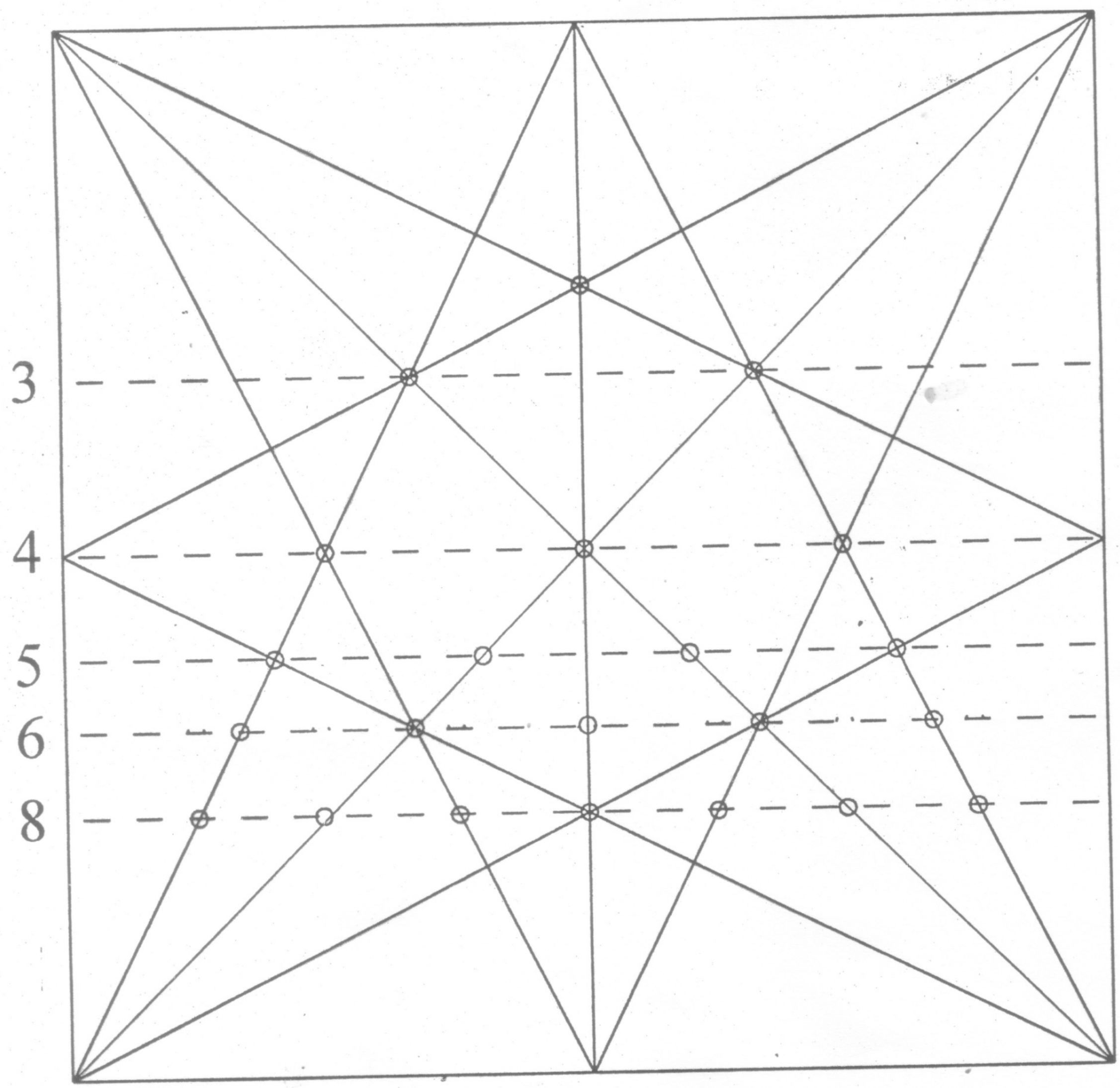
5. Repeat the process until the final square is the desired size.

6. The "spiral" shape becomes visible when the triangles are shaded as illustrated in the diagram.

Now the resulting figure is a Baravelle Spiral.



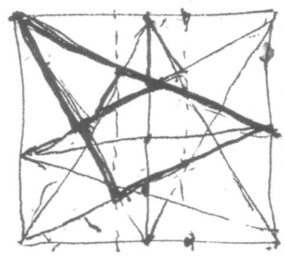


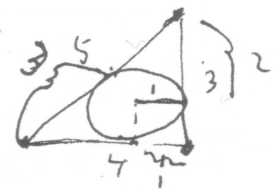

3
4
5
6
8

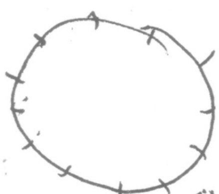
$\boxed{3, 4, 5}$

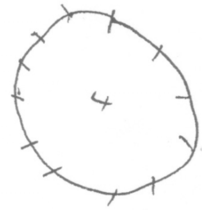
Kapusta

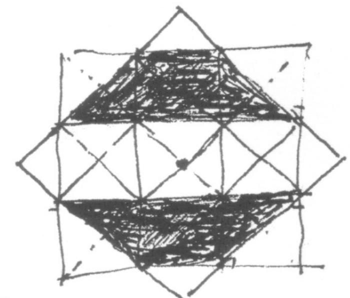
Gaudi





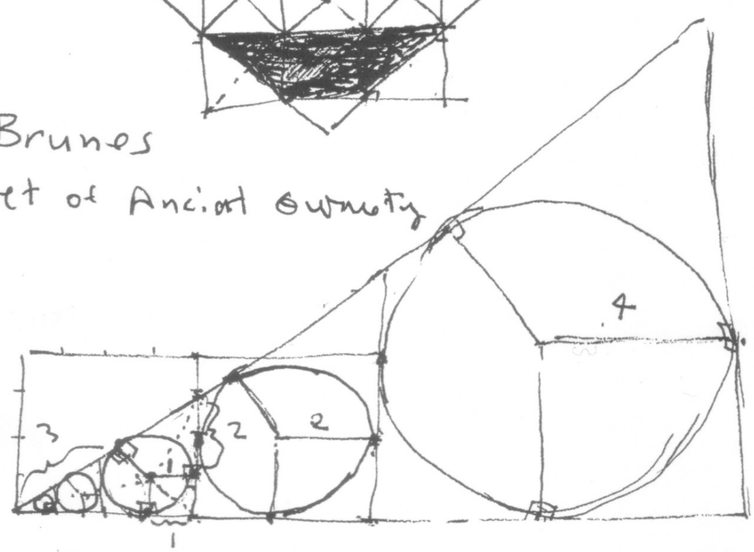






Ton Brunes

Secret of Ancient Geometry



Theory of Proportions

- 1. Modulor of Le Corbusier
- 2. Roman System of Proportions

The Modulor of Le Corbusier

- Blue $2/\phi$ 2 2ϕ $2\phi2$ $2\phi3$ $2\phi4$ $2\phi5$ …
- Red 1 ϕ $\phi2$ $\phi3$ $\phi4$ $\phi5$ $\phi6$
- …

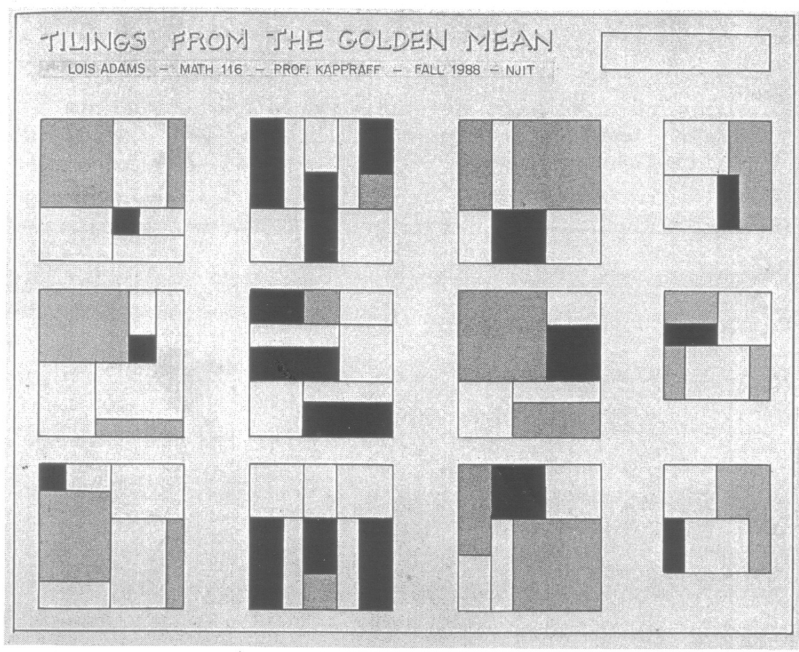
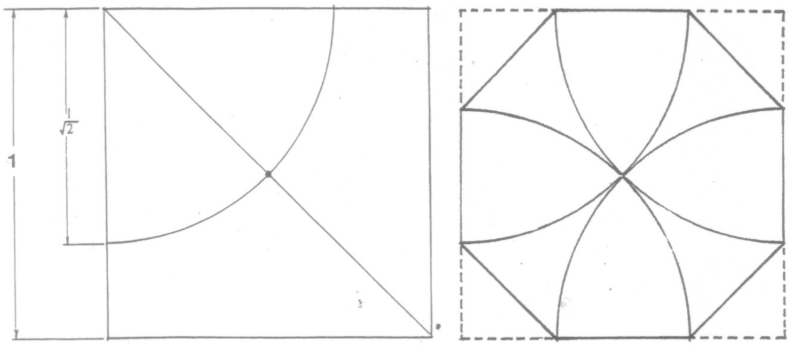


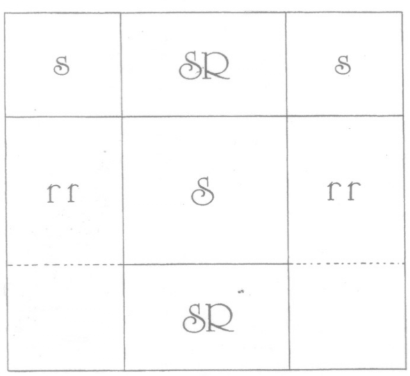
Figure 1.12 A Modulor tiling by Allison Baxter. In the first three columns a $2\phi^3$ by $2\phi^3$ square is subdivided into three different sets of of tilings. Each set uses the same tiles but is arranged in three different ways. The last column presents the tiling of a 5- by 5-inch square to within ¼-inch tolerance by the same tiles arranged in three different ways.

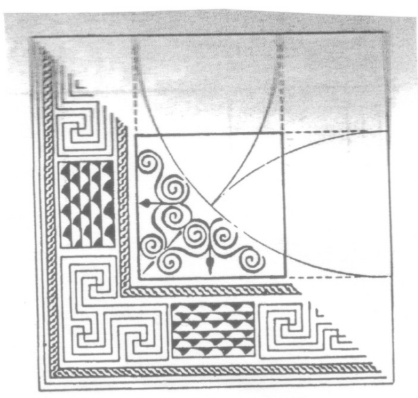
Unite’ House of Le Corbusier designed with the Modulor



a. b.

Figure 3. a) The sacred cut reduces a length by a factor of 1/ √2; b) four sacred cuts form the vertices of a regular octagon.





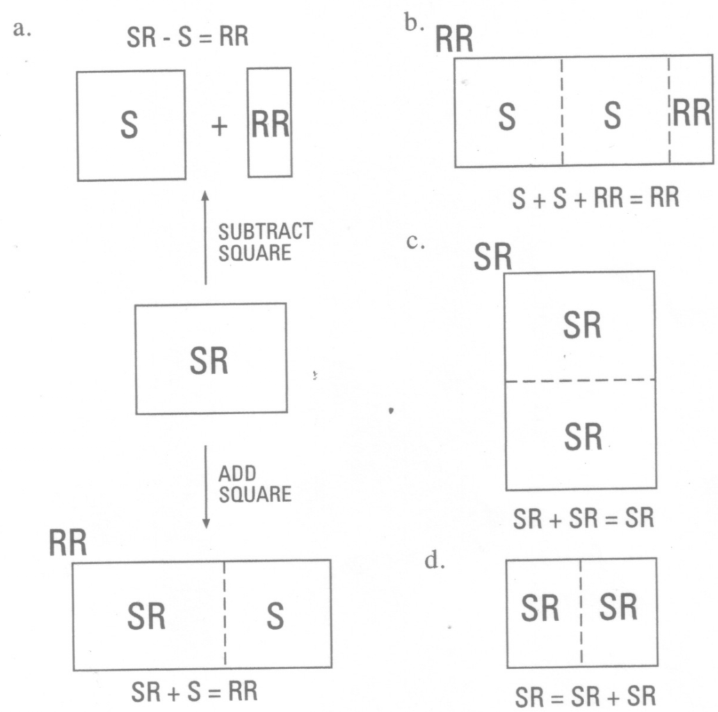
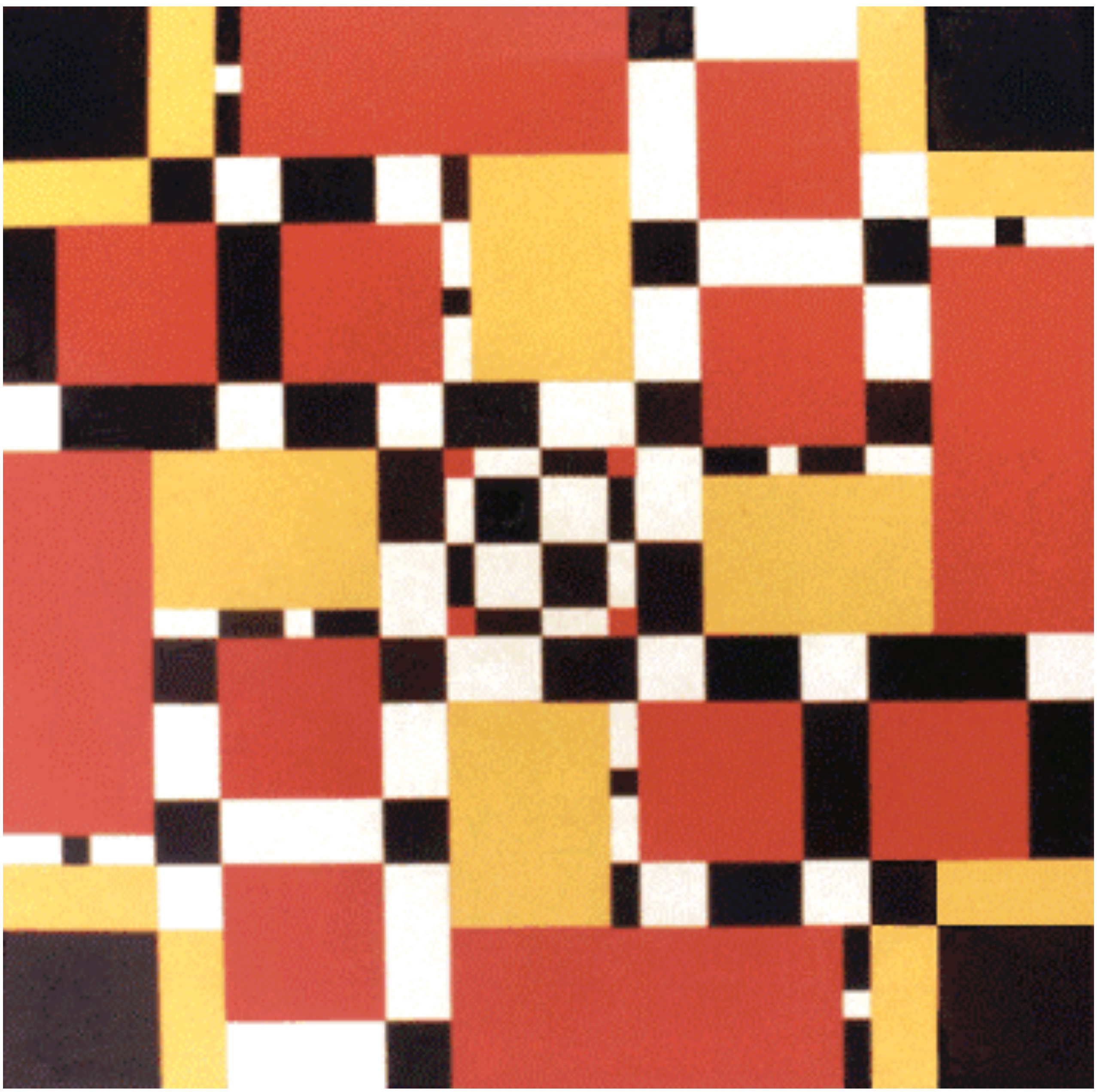


Figure 2. The square (S), square root rectangle (SR), and the Roman rectangle (RR) are interrelated.

Mosaic with rectangles from the Roman system at three scales



The System of Silver Means based on Pell's Series

-
-
- … $1/\theta$ 1 θ θ^2
- … $\sqrt{2}/\theta$ $\sqrt{2}$ $\theta\sqrt{2}$
- … $2/\theta$ 2 2θ $2\theta 2$

Tiling Patterns

- 1. Op-Tiles
- 2. Truchet Tiles
- 3. Kufi Tiles
- 4. Labyriths
- 5. Sona Sand Drawings
- 6. Lunda Patterns
- 7. Penrose - Islamic Tilings
- 8. LatticeTilings

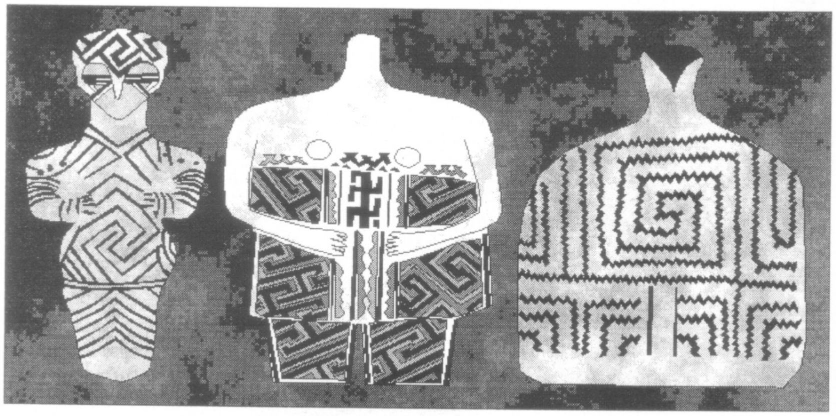
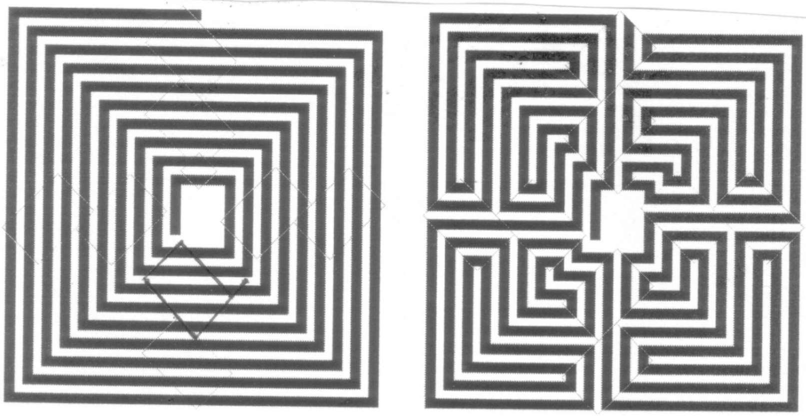
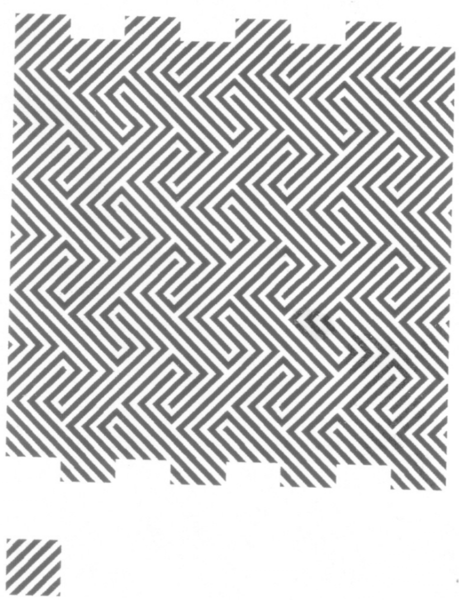


Figure 17. Neoltihic artefacts from Vincha (Yugoslavia), Tisza (Hungary) and Vadastra (Romania).



(a) (b)

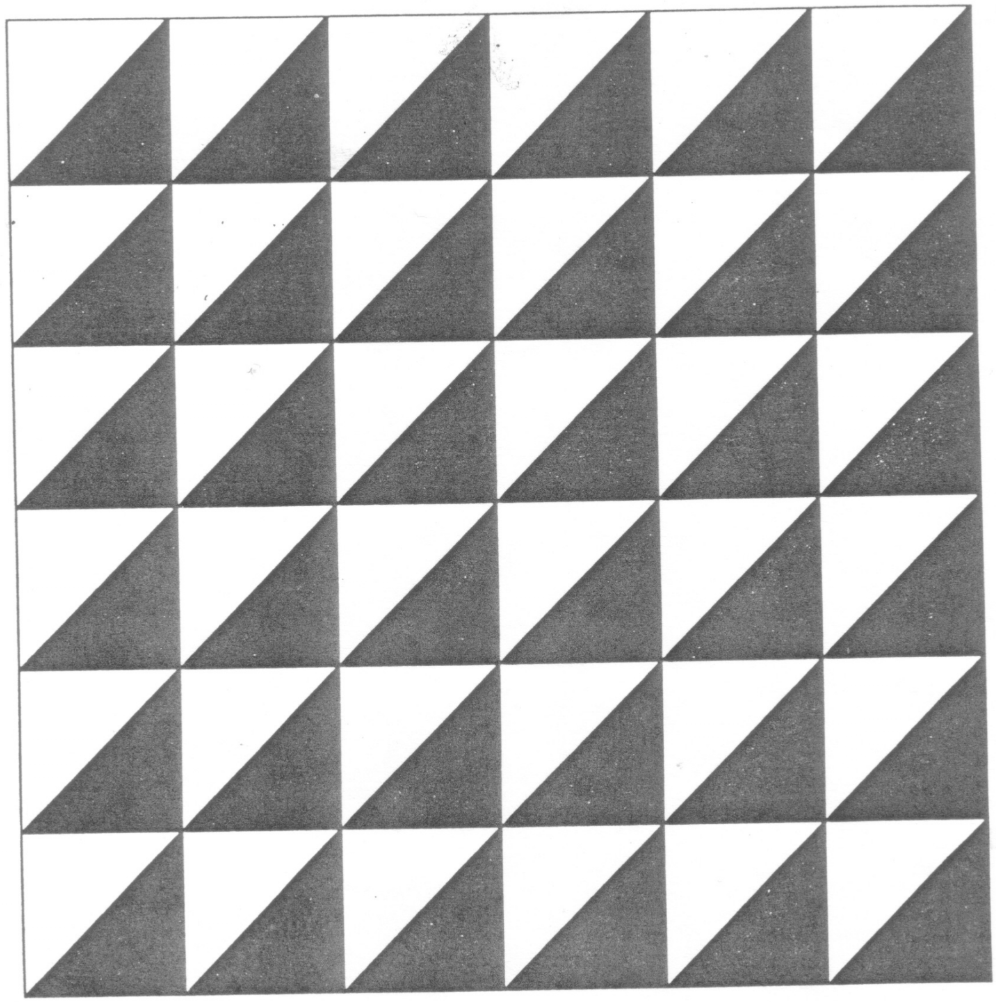
Figure 7. Roman maze (b) obtained from a meander spiral (b).

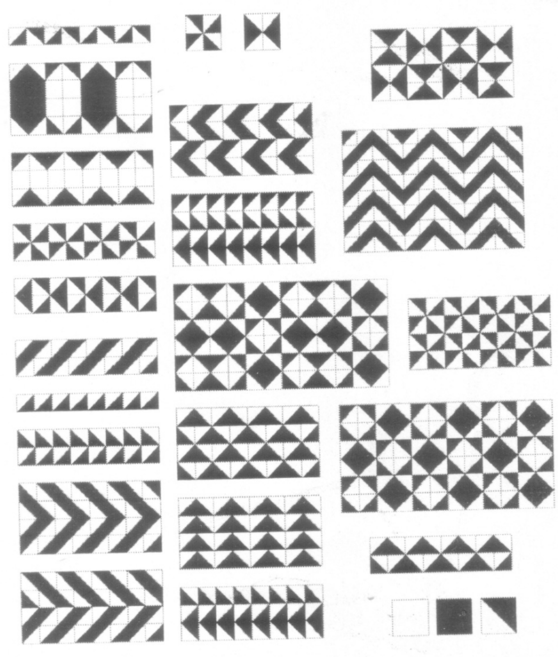


OP-TILES

Figure 19. Key-pattern created from a single Op-tile.

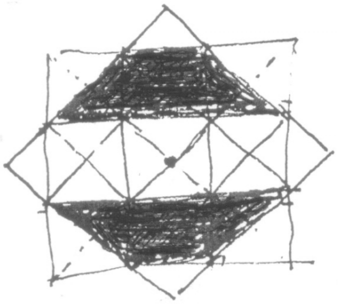
TRUCHET TILES

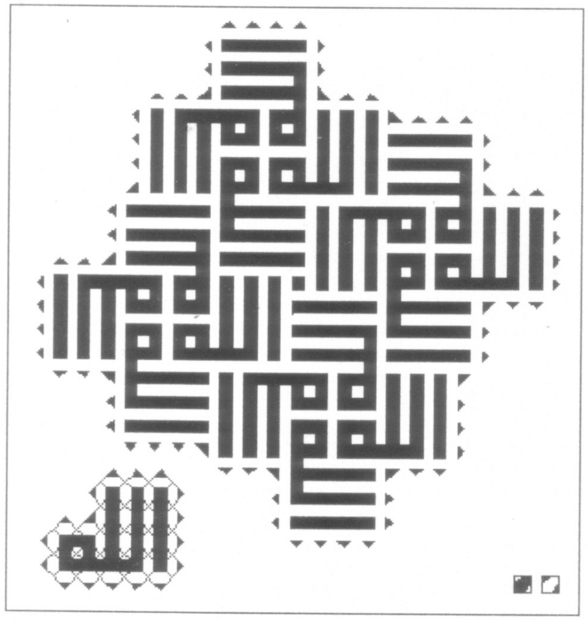




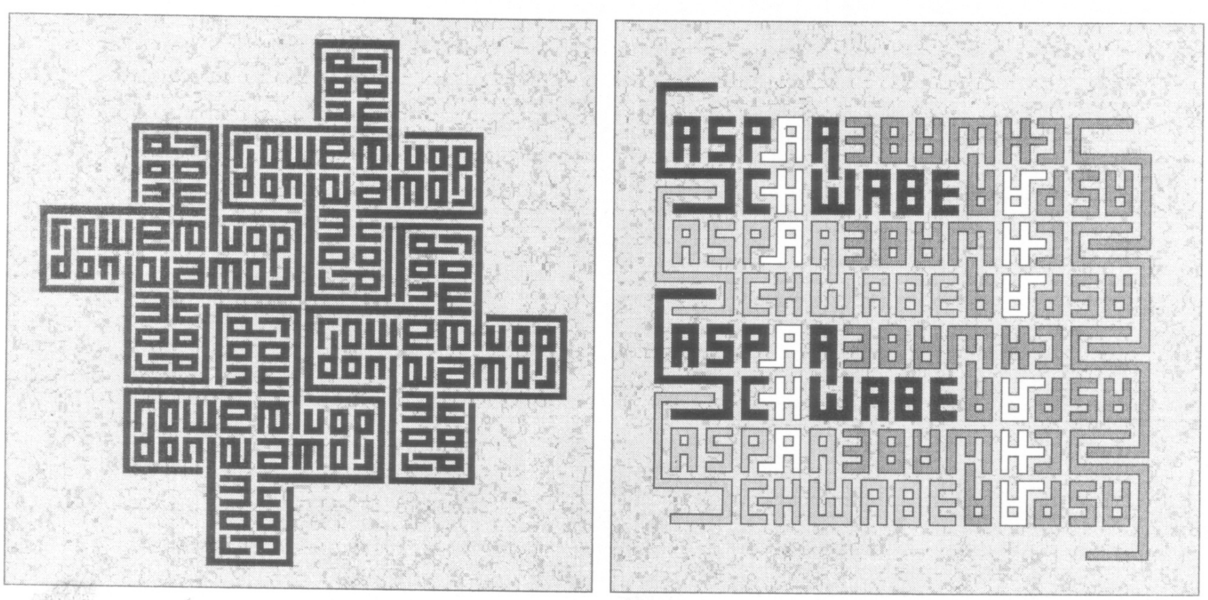
Truchet tile, the complete set of antisymmetry tiles and Neolithic patterns derived from it.

KUFI TILES





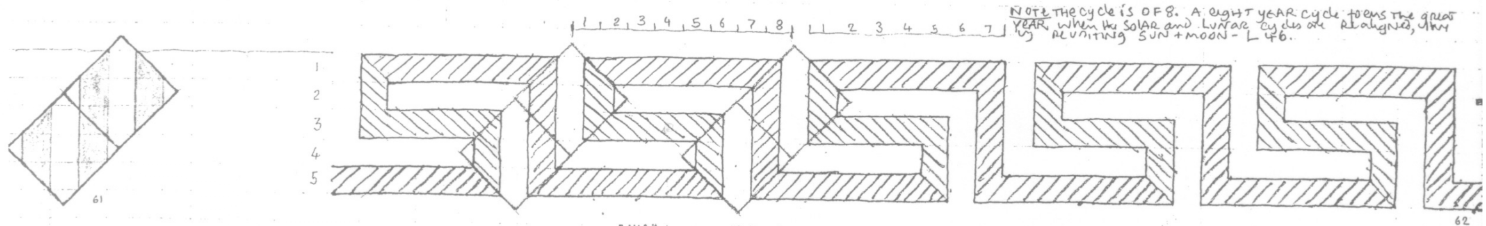
"Allah" in a Kufic script, Barda, Azerbaijan, XIV century (see, K.S.Mamedov: *Crystallographic Patterns*, Comp. & Maths. with Appl., **12B**, 3/4 (1986), 511-529).



Kufic scripts designed by the author.

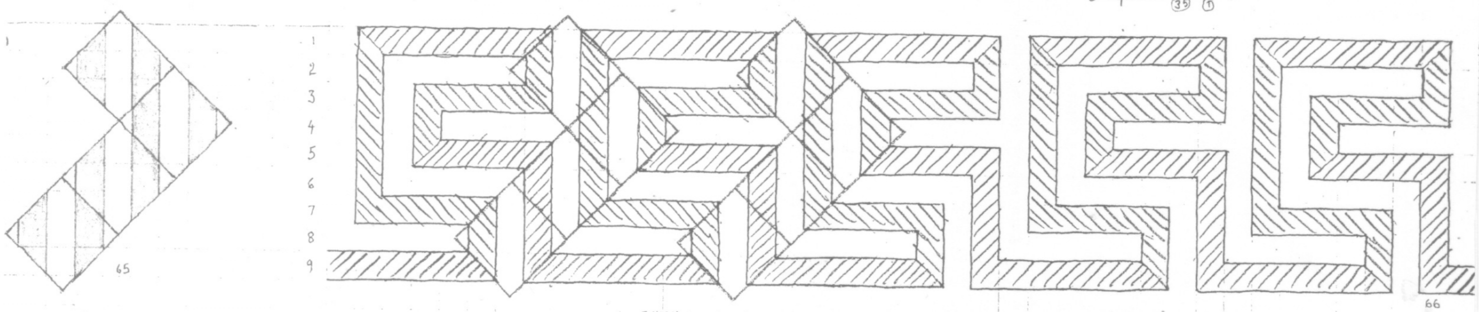
LABYRINTHS ANALYZED BY BEN NICHOLSON

NOTE THE cycle is OF 8. A eight year cycle forms the great YEAR, when the SOLAR and LUNAR cycles are REaligned, thus REUNITING SUN + MOON - L 46.



TWO TURN MEANDER, INTROVERT, 2V+1 H CHIP, 5 HIGH BASE RIGHT

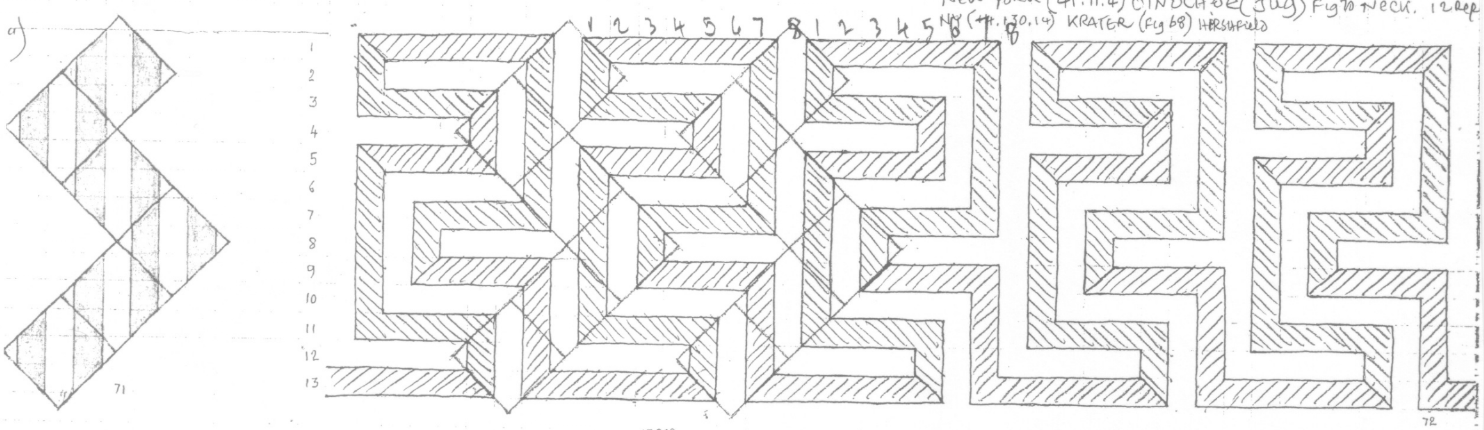
ATHENS 811, 2+½ H chip; ATHENS 804, 2V+1H CHIP;
Note That
NY (41.11.4) (Fig 64) Belly.
35 REPEATS (35) (1) ✓



THREE TURN MEANDER, INTROVERT, 5 HIGH 4V+1H CHIP, ; BASE RIGHT, UP.

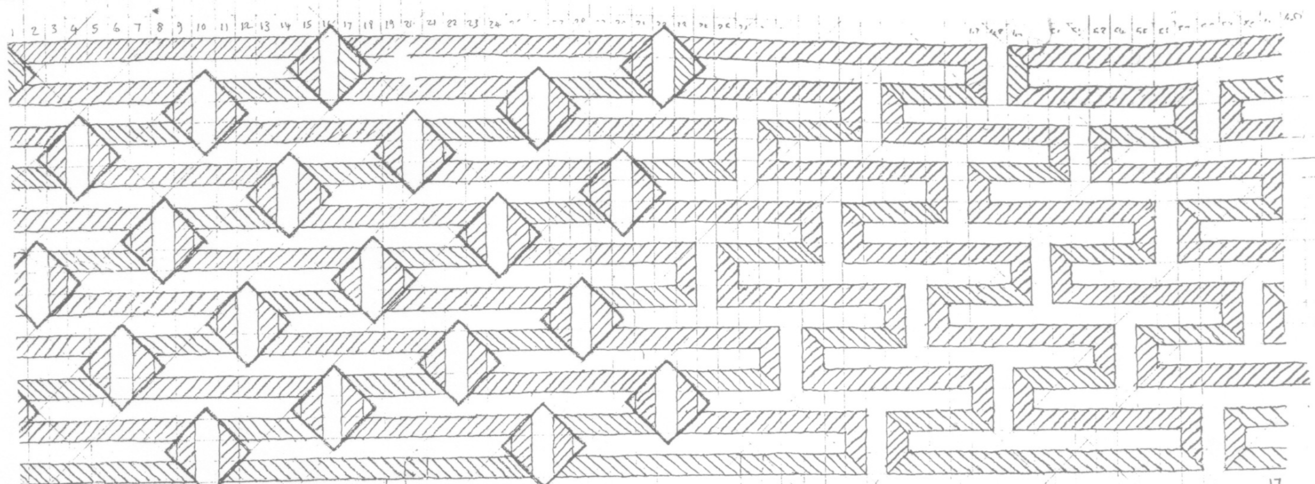
34 (1) 34 REPEATS.

ATHENS 804 (GGP. PL 6); ATHENS 18432 (GGP. PL 12); PARIS A552.
ATHENS 990 (BLU); NEW YORK HIRSHFIELD (BLU) (SA Fig 410)
PARIS A 519 (GG. Fg. 336),
NEW YORK (41.11.4) OINOCHOE (JUG) Fig 70 NECK. 12 Rep
NY (44.130.14) KRATER (Fig 68) HIRSHFIELD



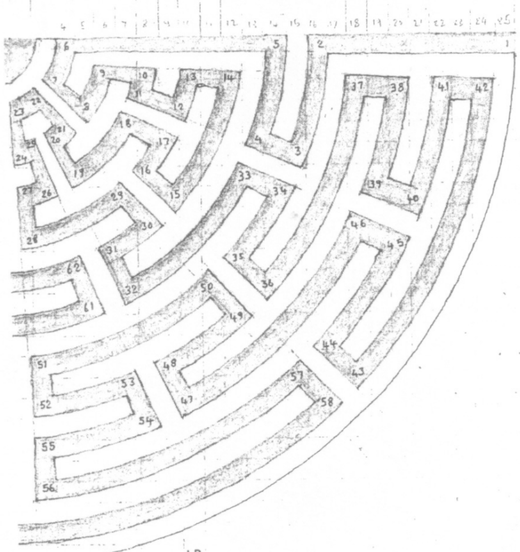
FOUR TURN MEANDER, INTROVERT, 13 BAR 6V+1H chip, BASE RIGHT, UP

ATHENS 804 (GGP. PL 6); PARIS A517 (GGP PL. 7a) BLU;
ATHENS



17

…s is nothing more than a staggered battlement



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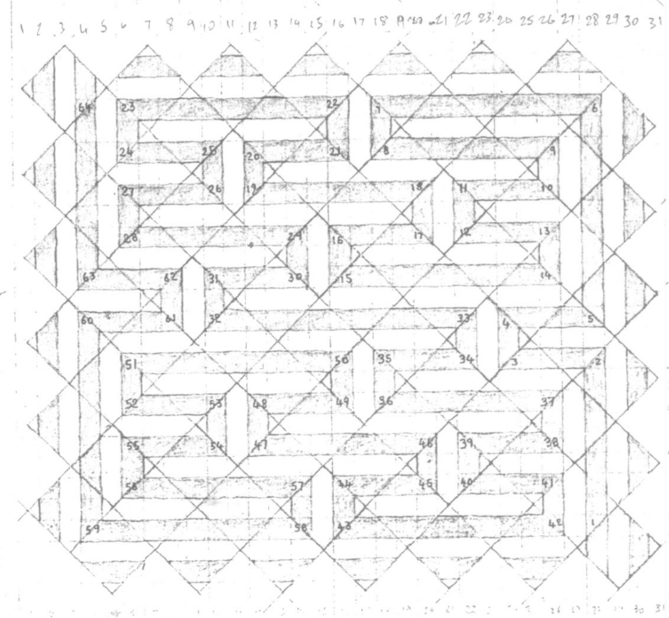
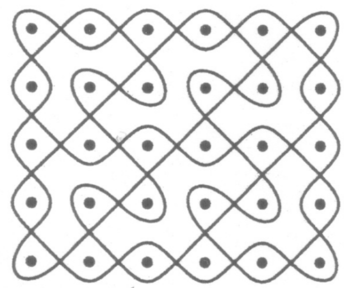
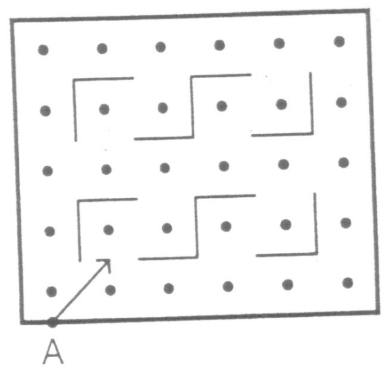


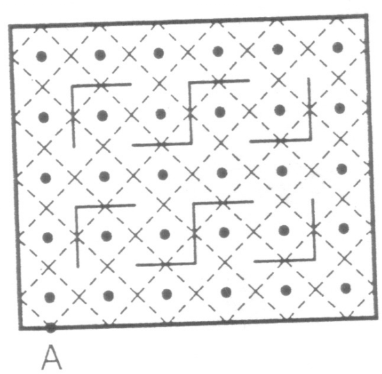
19

ON SONA SAND DRAWINGS, MIRROR CURVES AND THE GENERATION OF LUNDA-DESIGNS

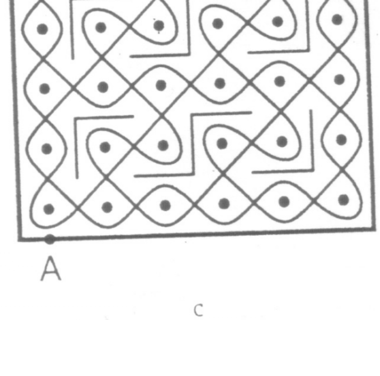




a



b



c

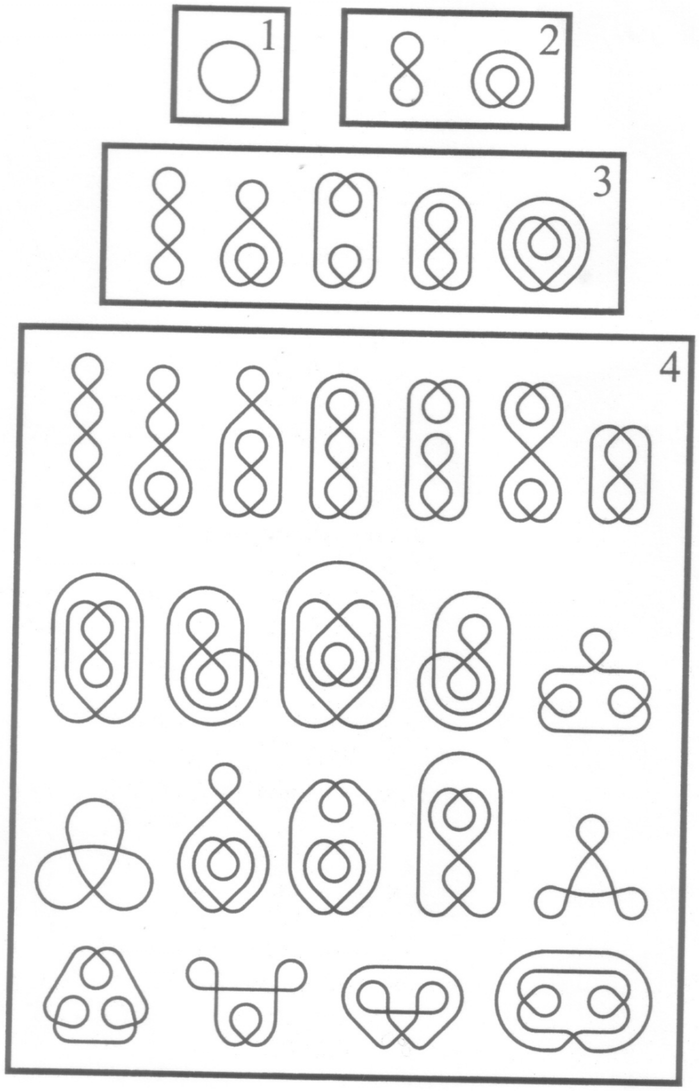
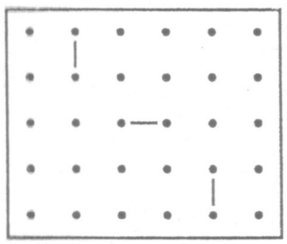
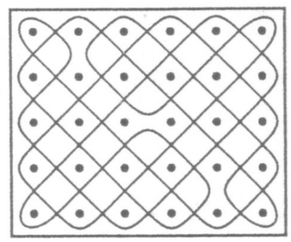


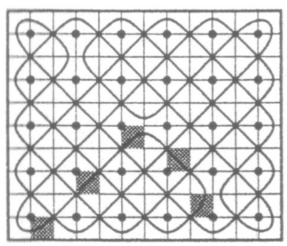
Figure 3: *All sona maps on* n *dots for* n *between* 1 *and* 4.



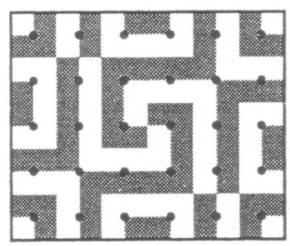
Mirror design
a



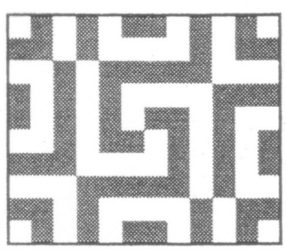
Corresponding mirror curve
b



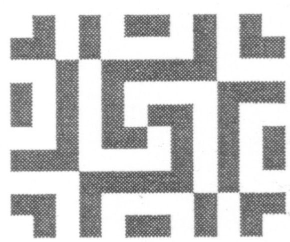
Starting the colouring
c



Final black-and-white pattern
(with the grid points marked)
d

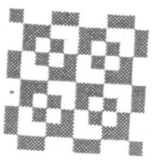


Final black-and-white pattern
(with the grid points unmarked)
e



Final black-and-white pattern
(with the border rectangle unmarked)
f

Example of a black-and-white colouring



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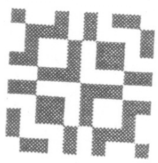

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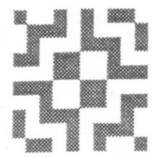

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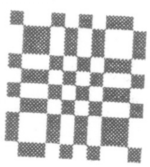

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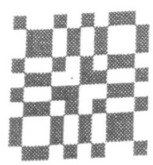

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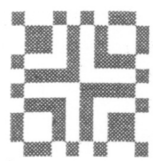

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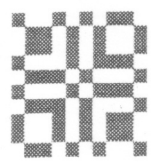

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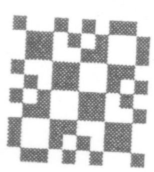

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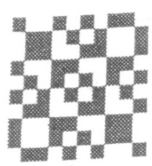

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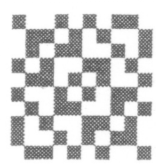

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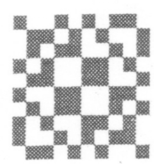

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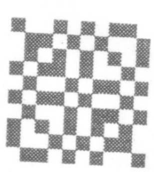

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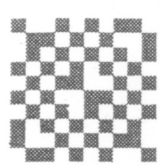

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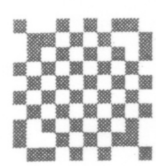

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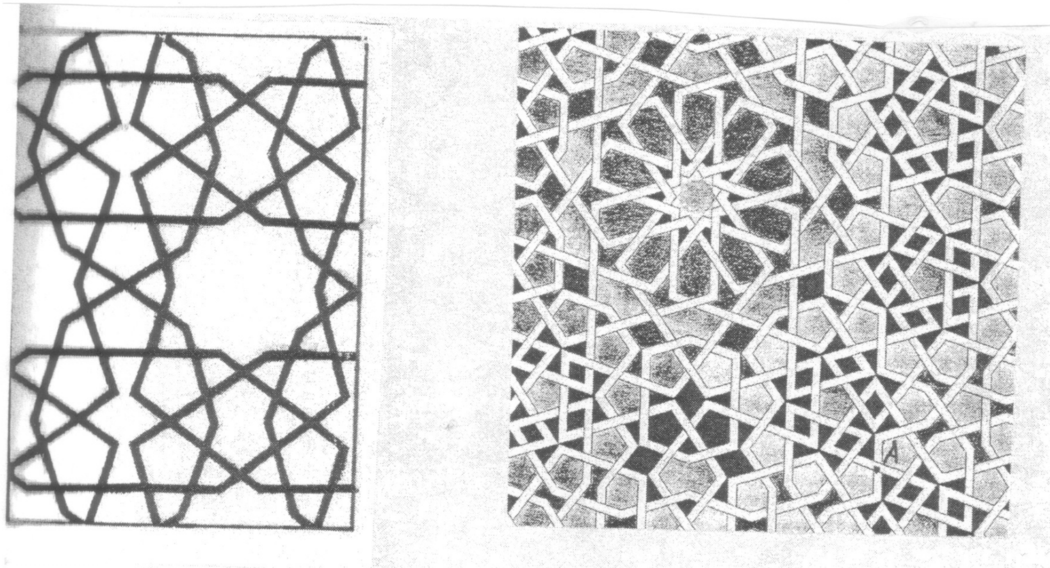
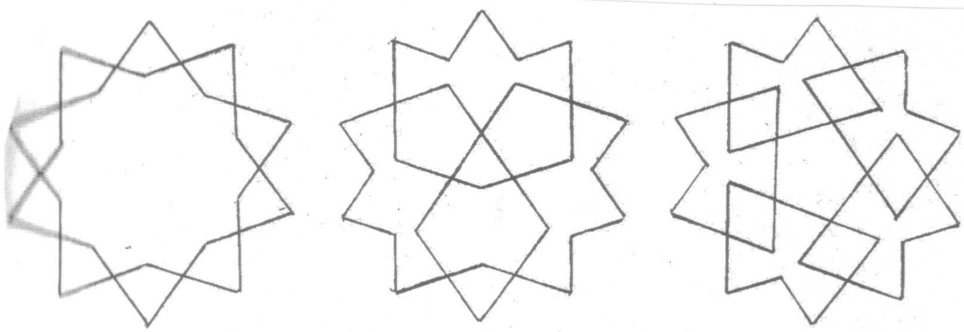


(a) (b)

Figure1. *Interlacing patterns from Afghanistan and Turkey.*



Star Lion head Five-diamond

. Skeletal forms of motifs from Figure 1.

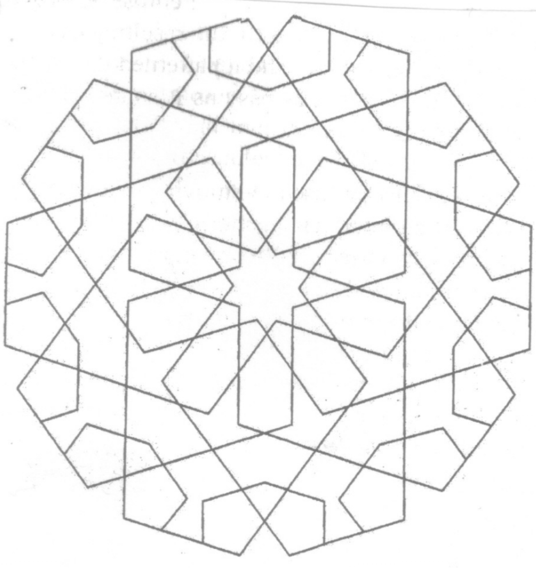


Figure 4. *The skeletal form of the rosette motif.*

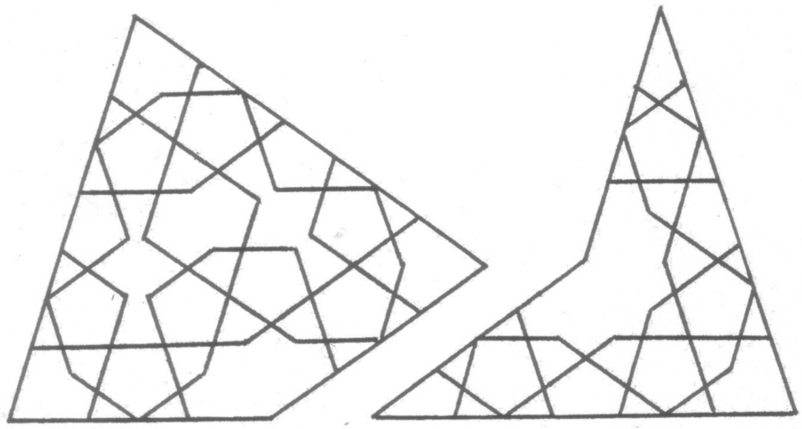


Figure 5. *A patterned kite and dart.*

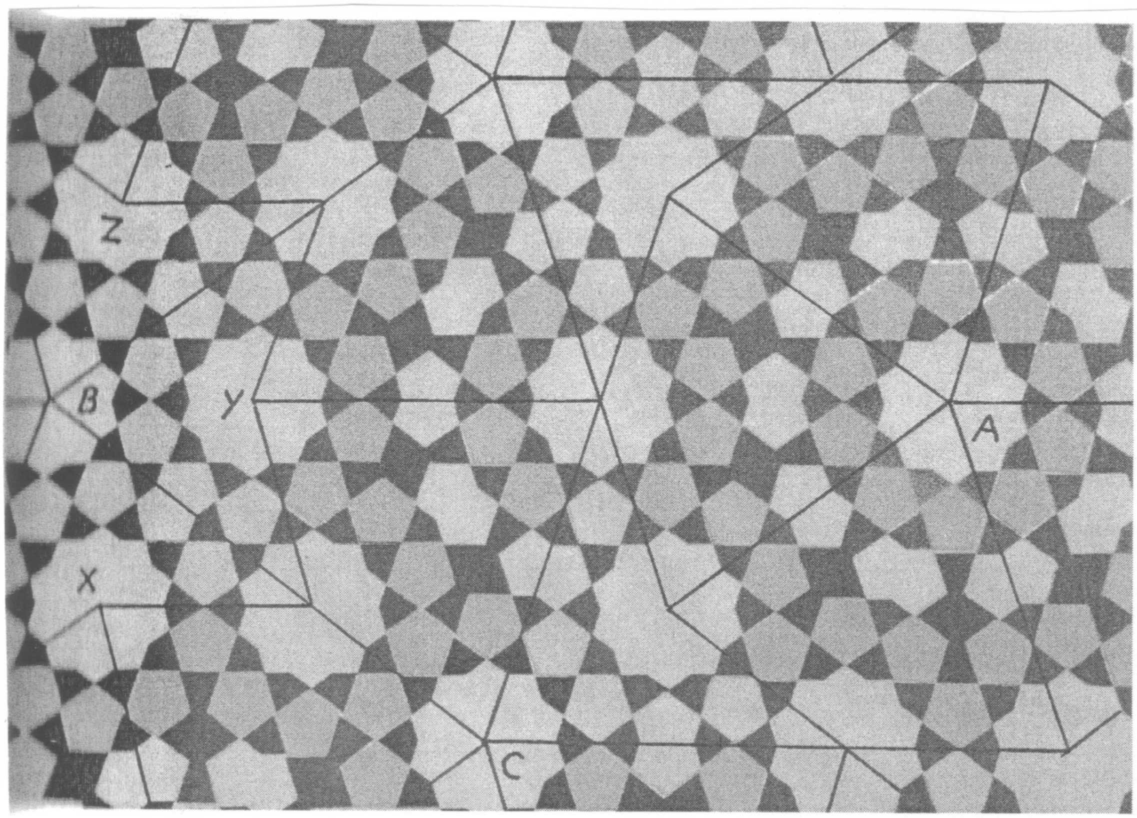
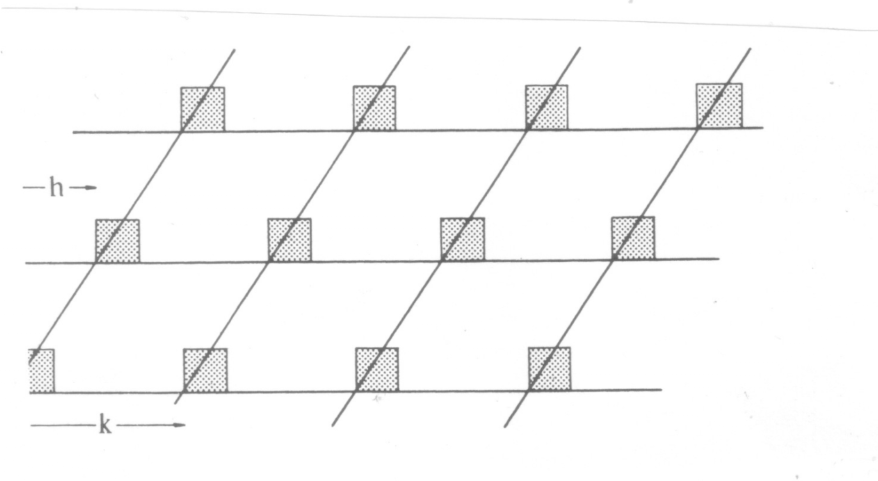
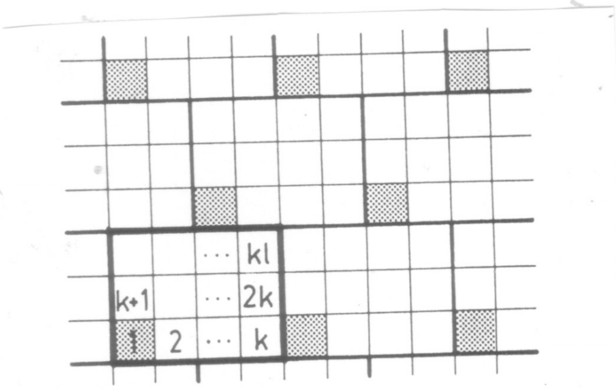
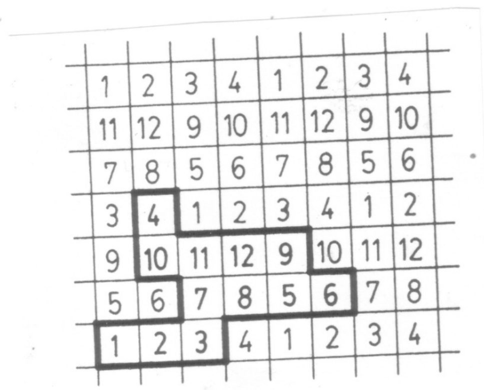


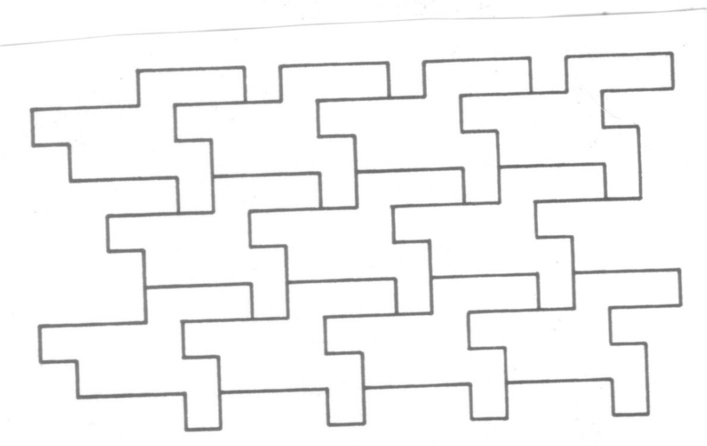
Figure 6. *A basic Penrose-type pattern obtained from Figure 5.*

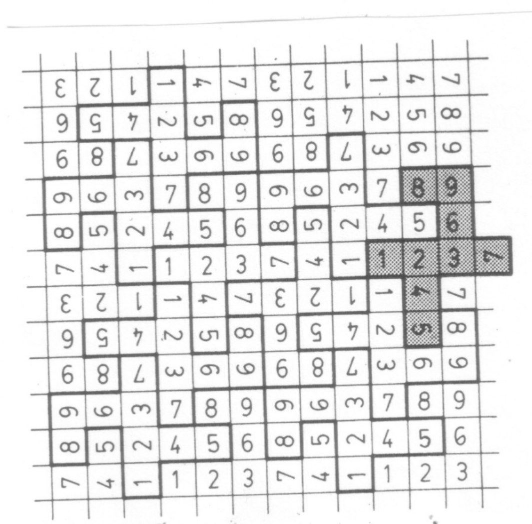
LATTICE TILINGS



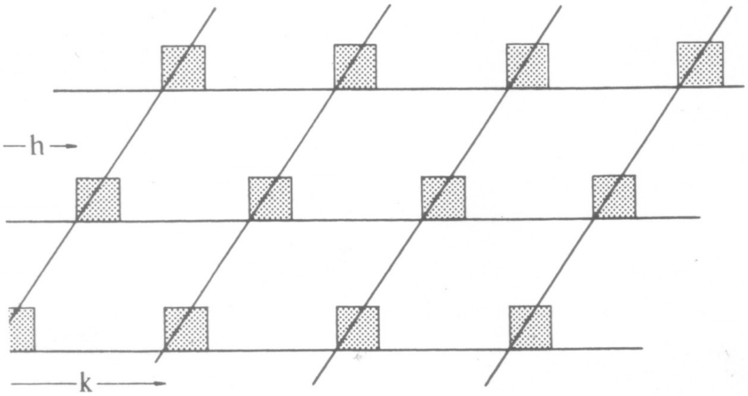


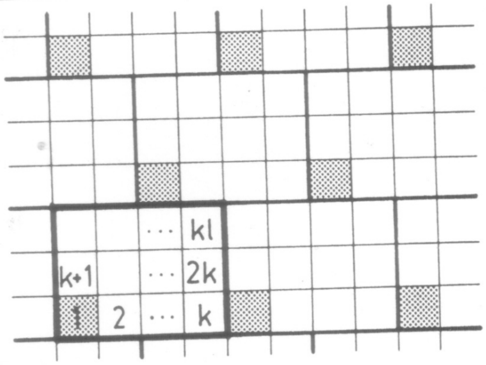


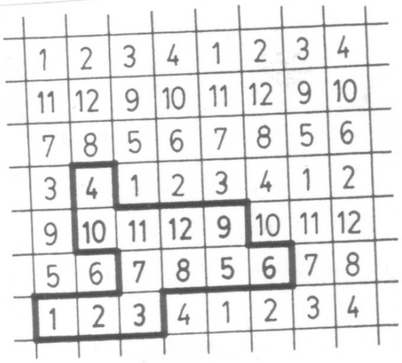


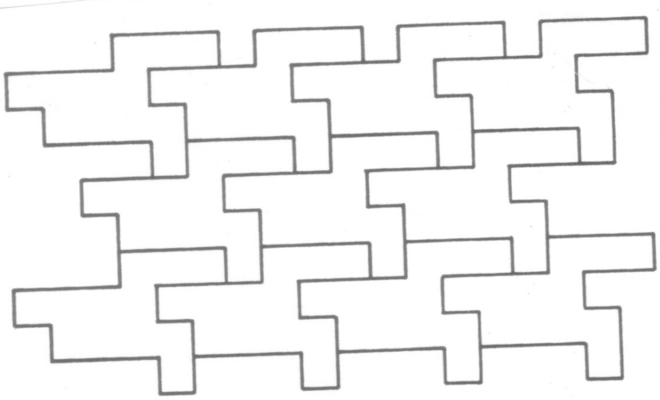


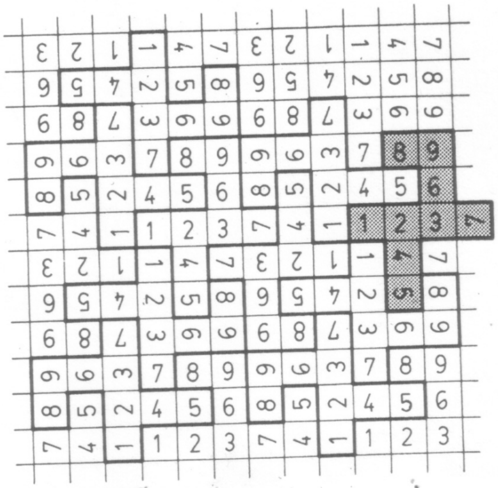
LATTICE TILINGS











12	7	8	9	10	11	12	7
6	1	2	3	4	5	6	1
10	11	12	7	8	9	10	11

3	4	5	6	1	2	3	4
7	8	9	10	11	12	7	8
1	2	3	4	5	6	1	2

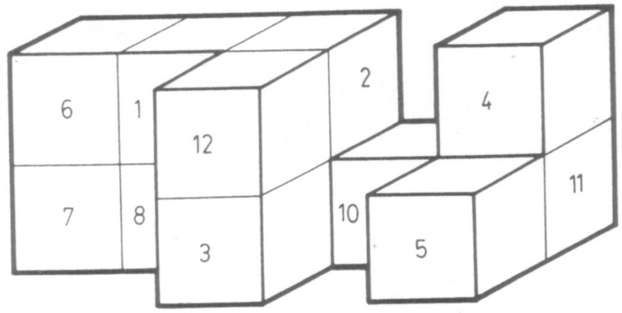


Figure 10. Two consecutive levels of the grid determined by the parameters k = 6, l = 2, m = 1, h = 4, i = 1, j = 1 and a spacefilling constructed from this grid ● Deux niveaux consécutifs de la grille déterminée par les paramètres k = 6, l = 2, m = 1, h = 4, i = 1, j = 1 et un polyèdre remplissant l'espace construit à partir de cette grille.

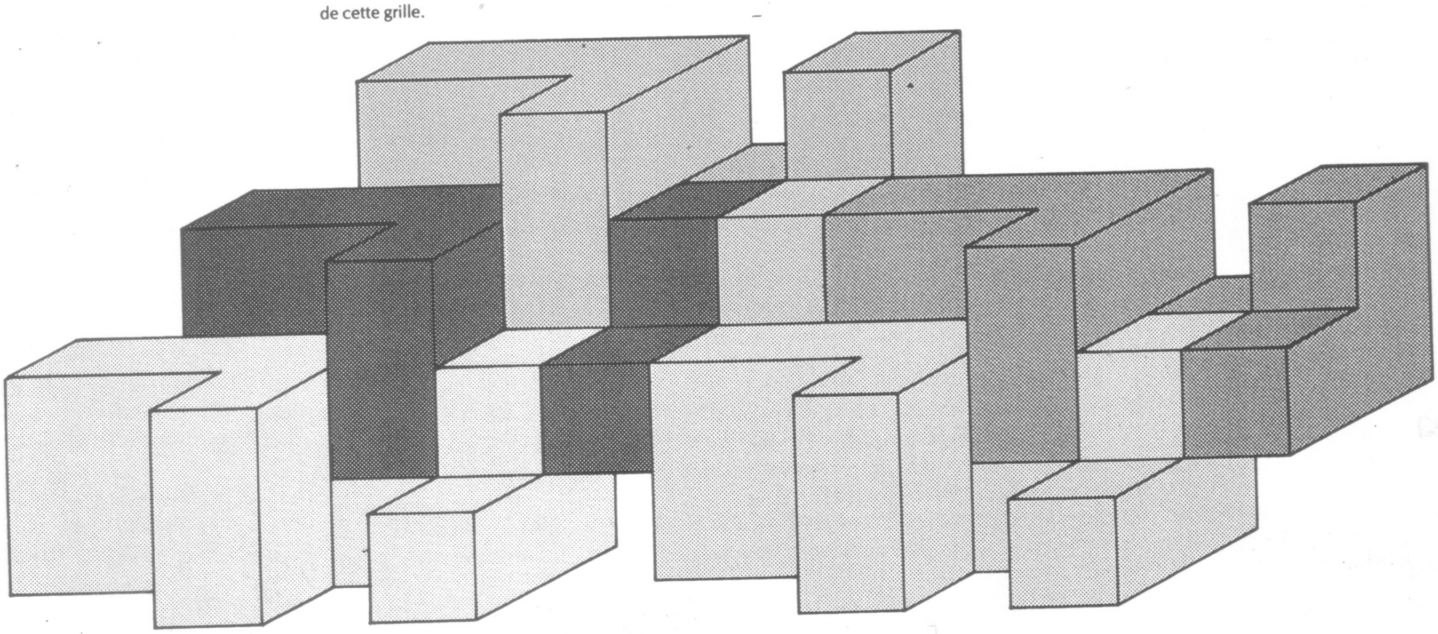


Figure 11. Five replicas of the spacefiller in position in three dimensional space ●

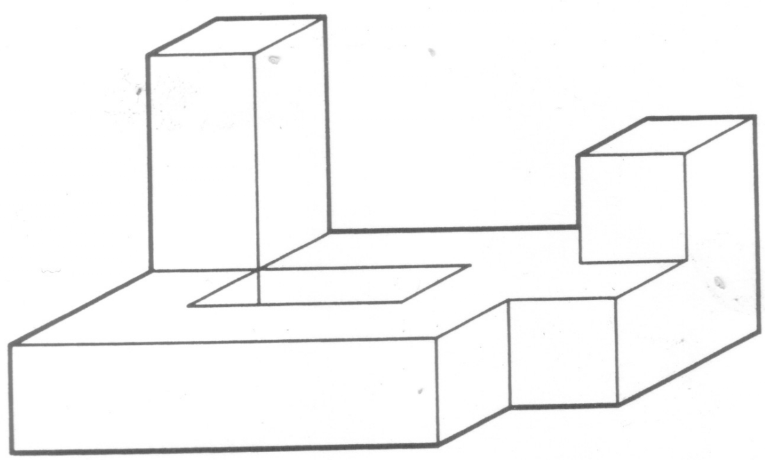
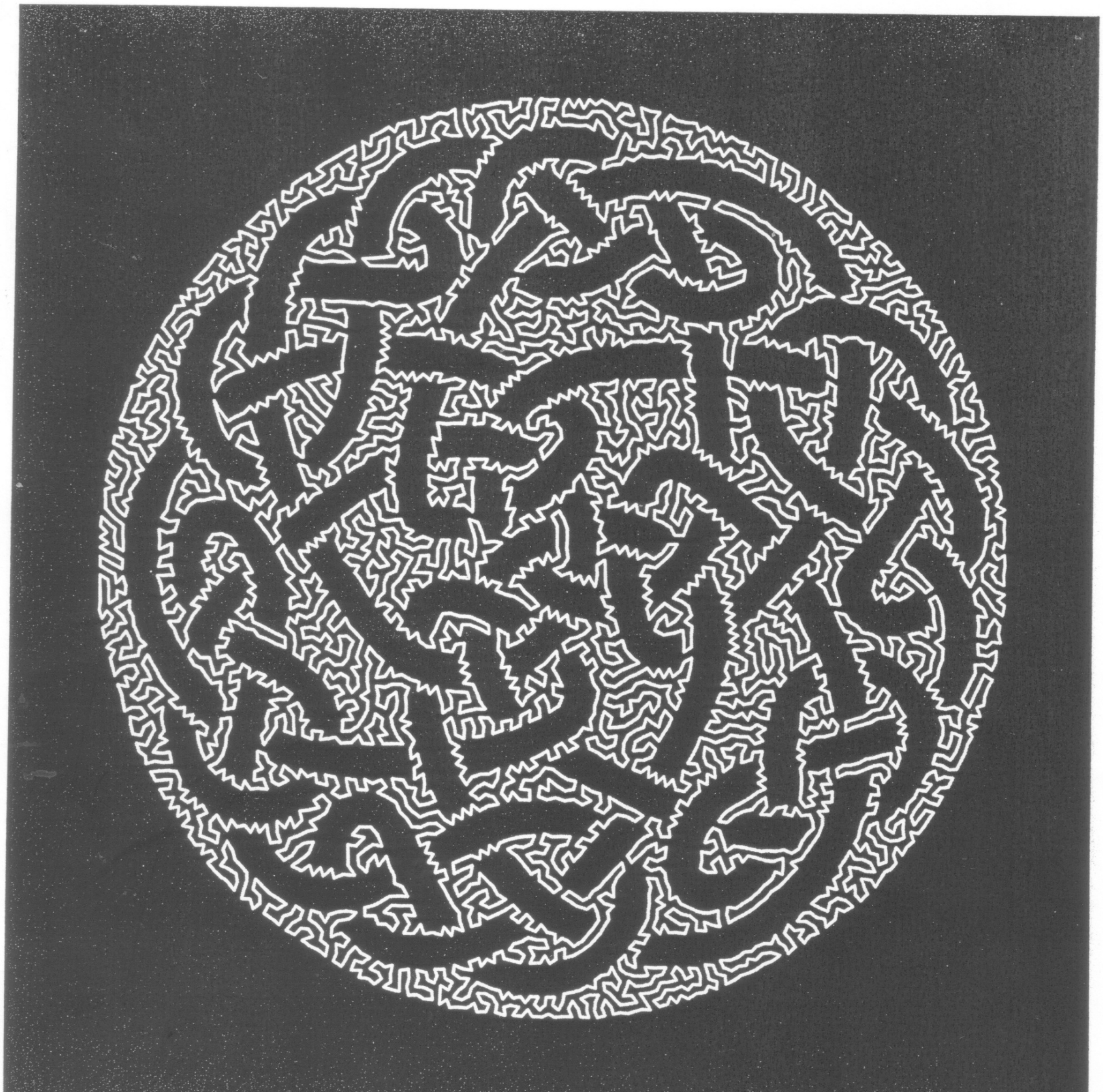


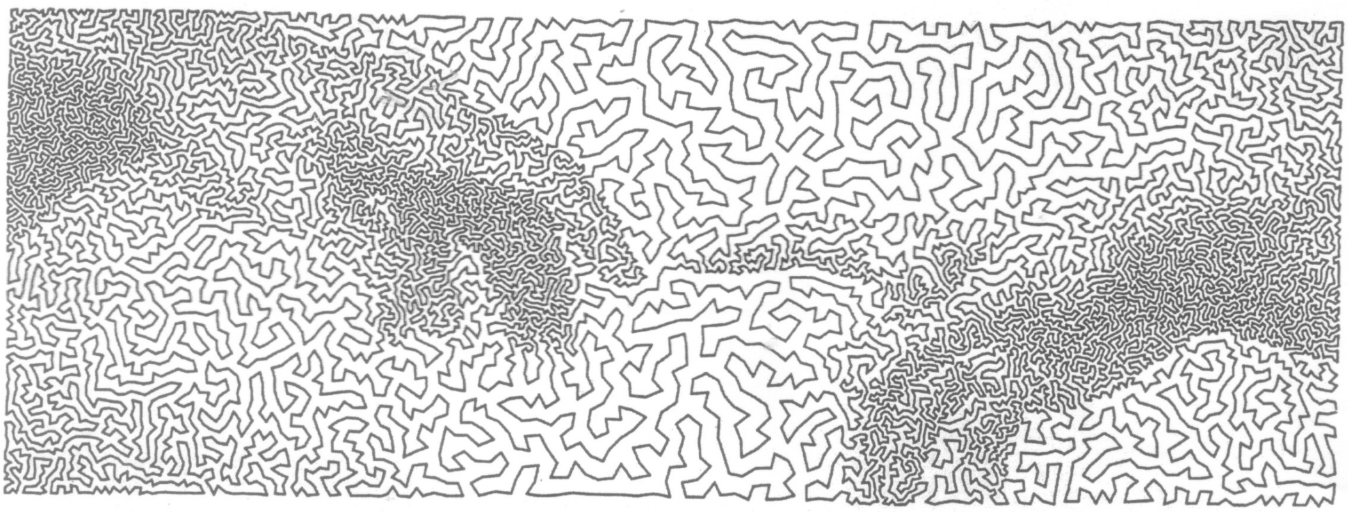
Figure 12. A spacefiller derived from the parameters k = 5, l = 3, m = 1, h = 1, i = 3 and j = 1. The plans of the three levels are shown at the left. In fact in this example the value of h is not used in the construction of the spacefiller but only determines how the different spacefillers fit together ●

Traveling Salesman Designs

- Tilings based on approximate Hamilton paths
- by
- Robert Bosch



Robert Bosch, *Knot?*, 2006
TSP Art continuous line drawing
117 ft curve on 34 × 34 in canvas

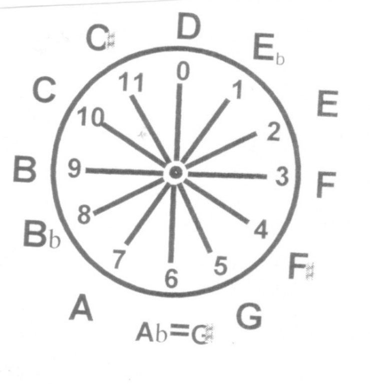


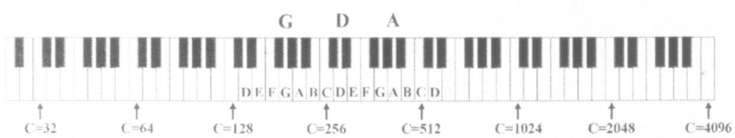
Robert Bosch, *Hands*, 2006 (after Michelangelo)
TSP Art continuous line drawing
188 ft curve on 44 × 19.5 in canvas

Symmetry and Music

- 1. Heptatonic scale
- 2. Pentatonic scale
- 3. African clapping patterns

SYMMETRY AND MUSIC





. The piano keyboard

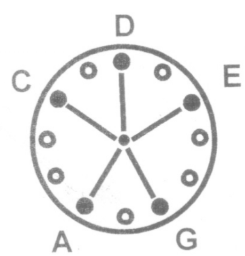


Figure 6. A pentatonic scale

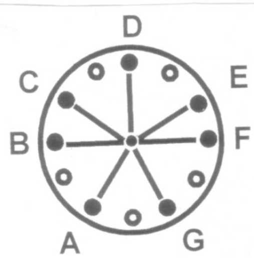


Figure 7. A heptatonic scale when inverted forms a pentatonic

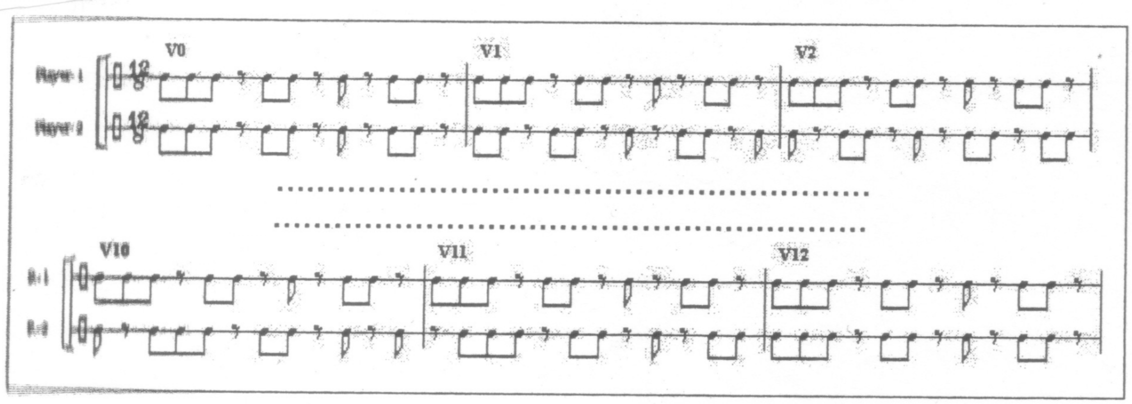


Figure 1: The first and last few bars of *Clapping Music.*

Nichomachus' Table

- Expansions of the ratio 3:2
- (as string lengths)

1	2	4	8	16	E	32	64	B
	3	6	12	24	A	48	96	E
		9	18	36	D	72	144	A
			27	54	G	108	216	D
				81	C	162	324	G
						243	486	C
							729	F

Alberti’s Musical Proportions

-
- 1 2 4 8 16 …
- 3 6 12 24 …
- 9 18 36 …
- 27 …

Bi-symmetric matrices lead to generalizations of the golden mean

$$\text{sqrt} \begin{matrix} 3 & 2 \\ 2 & 3 \end{matrix} = \begin{matrix} \text{phi} & 1/\text{phi} \\ 1/\text{phi} & \text{phi} \end{matrix}$$

phi = golden mean

$$\begin{matrix} 3 & 2 \\ 2 & 3 \end{matrix} \times \begin{matrix} 3 & 2 \\ 2 & 3 \end{matrix} = \begin{matrix} 12 & 13 \\ 13 & 12 \end{matrix}$$

Where $5^2 + 12^2 = 13^2$

Sqrt2 to 7 places derived from the musical scale

$1,\overline{1},1,2 \rightarrow 1:1$

$1,1,2,2$

$2,2,4,4$

$2,\overline{3},3,4 \rightarrow 3:2$

$1,\frac{2}{3},\frac{3}{2},2$

$1,\frac{4}{3},\frac{3}{2},2$

$6,8,9,12$

$12,16,18,24$

$12,\overline{17},17,24 \rightarrow 17:12$

$1,\frac{12}{17},\frac{17}{12},2$

$1,\frac{24}{17},\frac{17}{12},2$

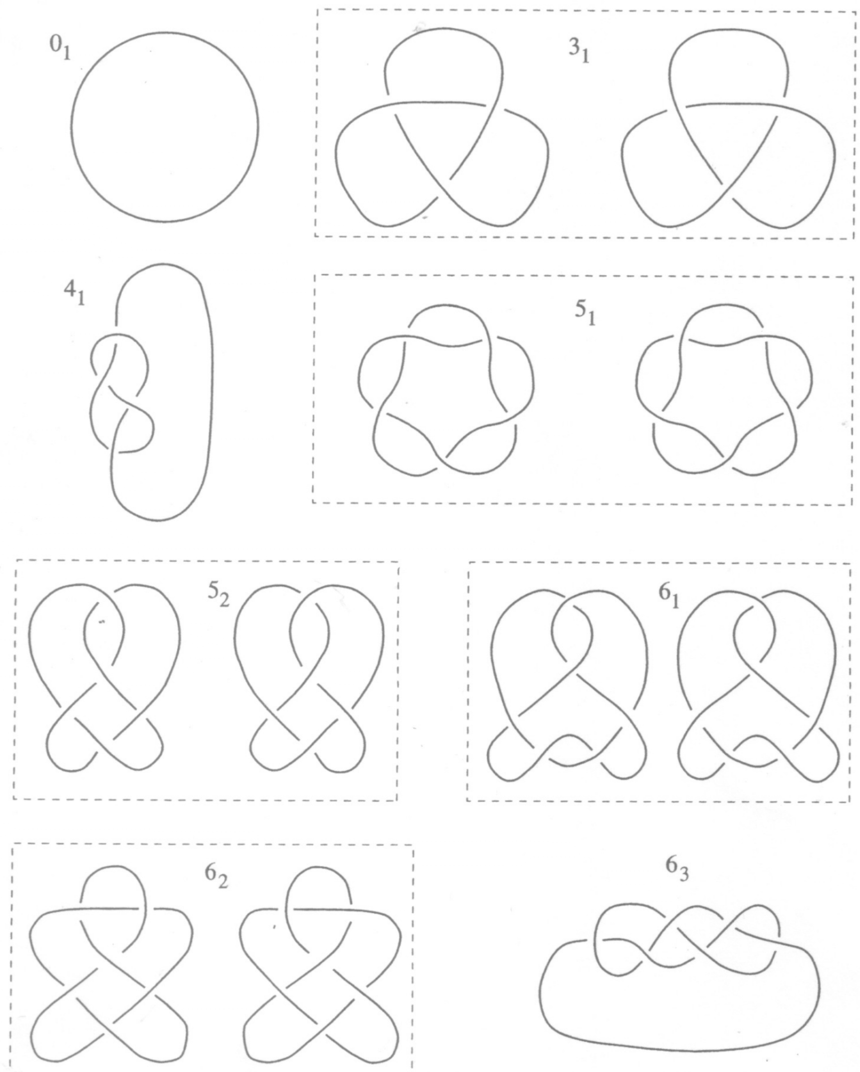
$204,288,289,408$

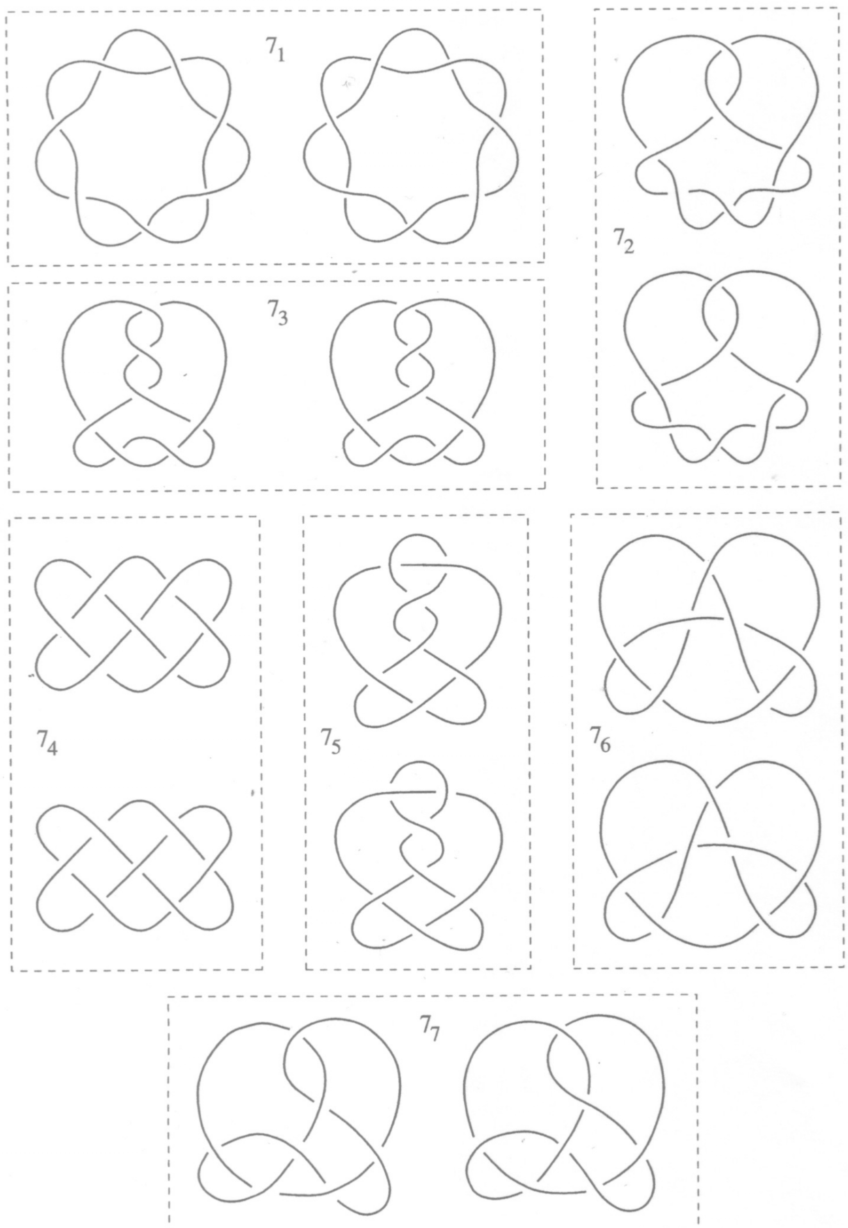
$408,566,568,816$

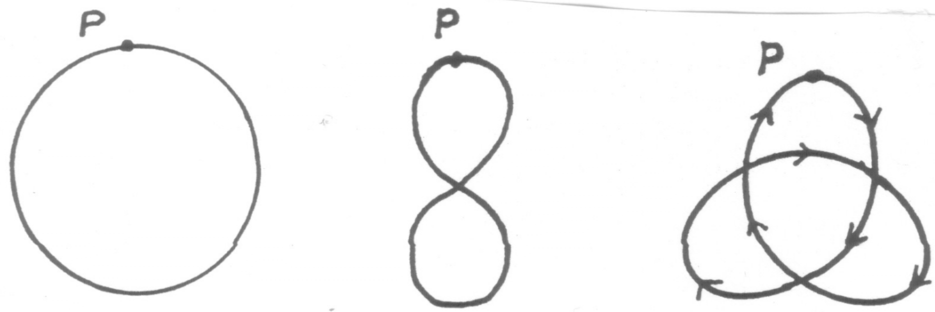
$408,\overline{577},577,816 \rightarrow 577:4$

Knot Theory

- 1. Knots up to 7 crossing
- 2. Curvos
- 3. Knots and surfaces


0_1
3_1
4_1
5_1
5_2
6_1
6_2
6_3

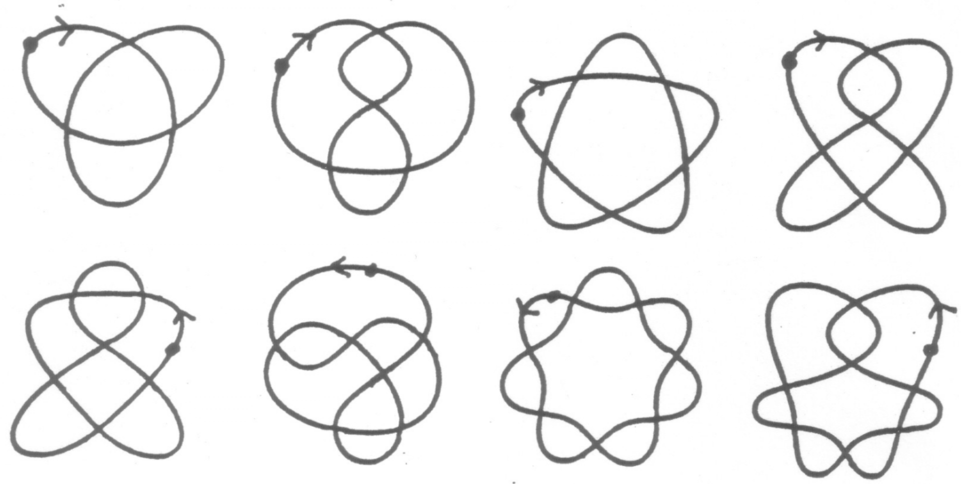

7_1
7_2
7_3
7_4
7_5
7_6
7_7

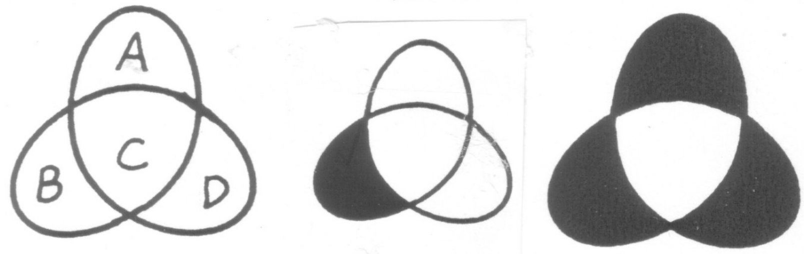


(a) (b) (c)

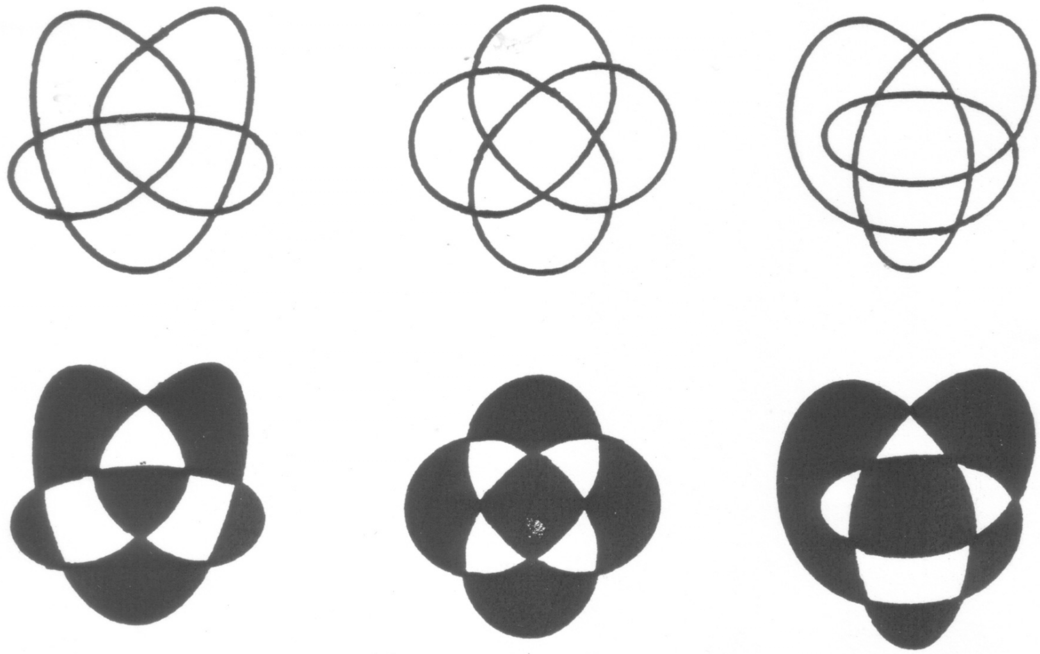
Figure 1.

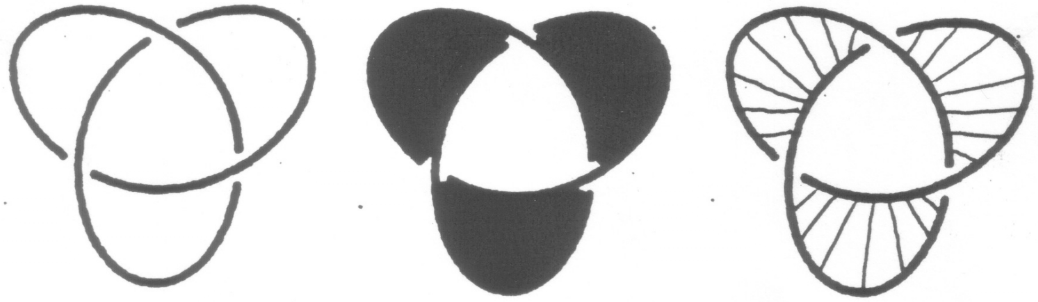
Additional examples of curvos are shown in Figure 2. In each case we can choose a starting point and a direction to traverse the curvo.





(a) (b)





(a) (b) (c)