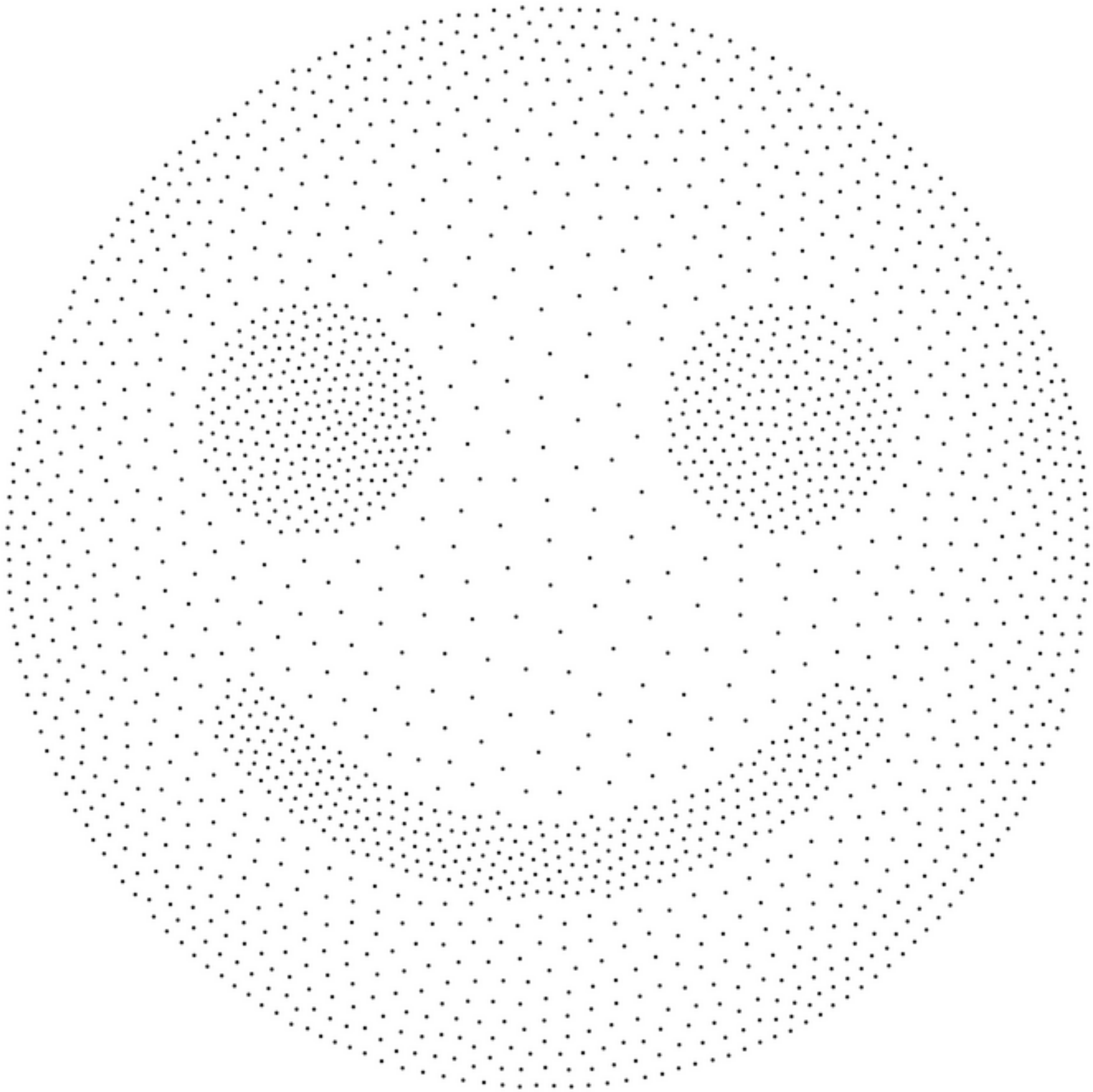
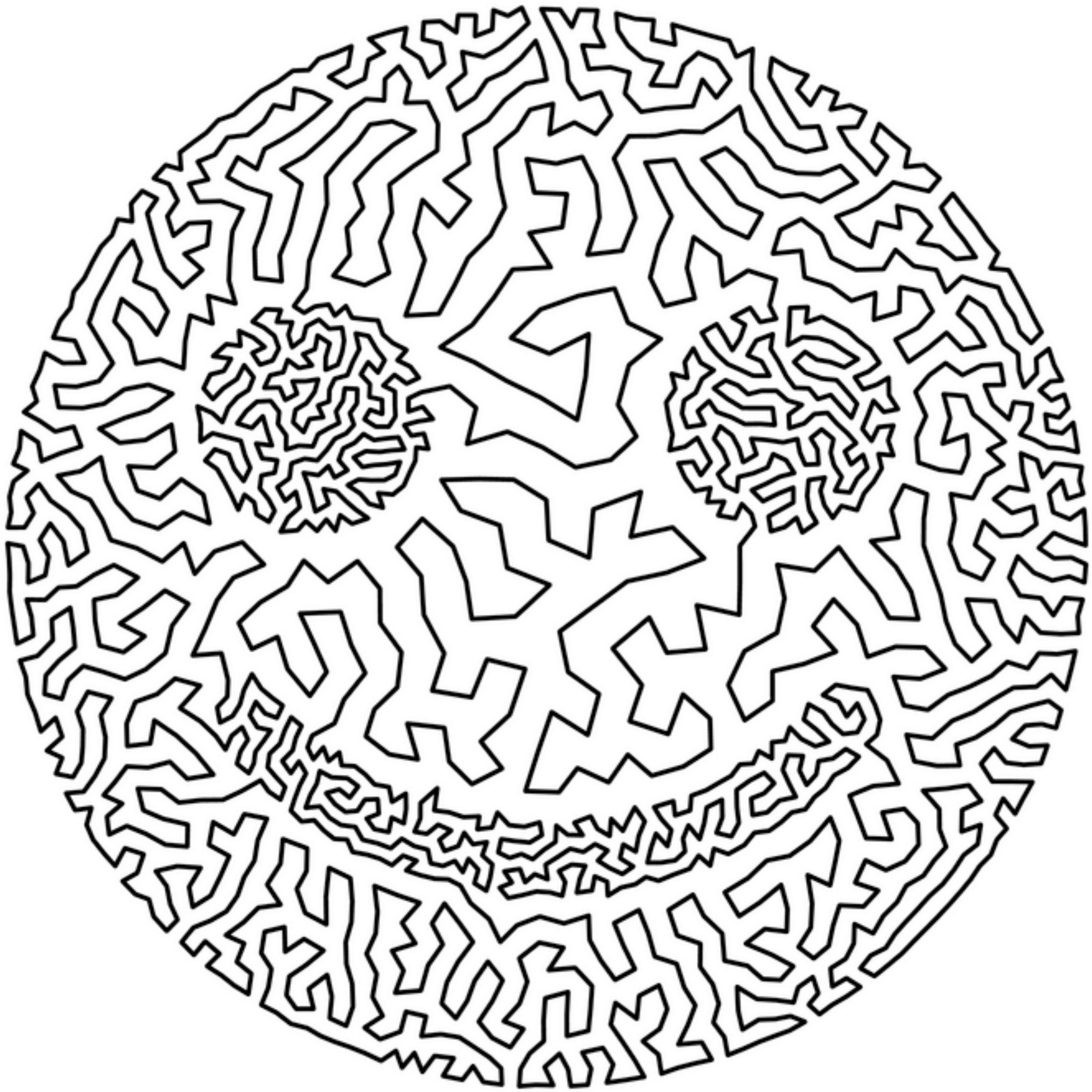
TSPortraits of Knots and Links

Robert Bosch

May 13, 2009

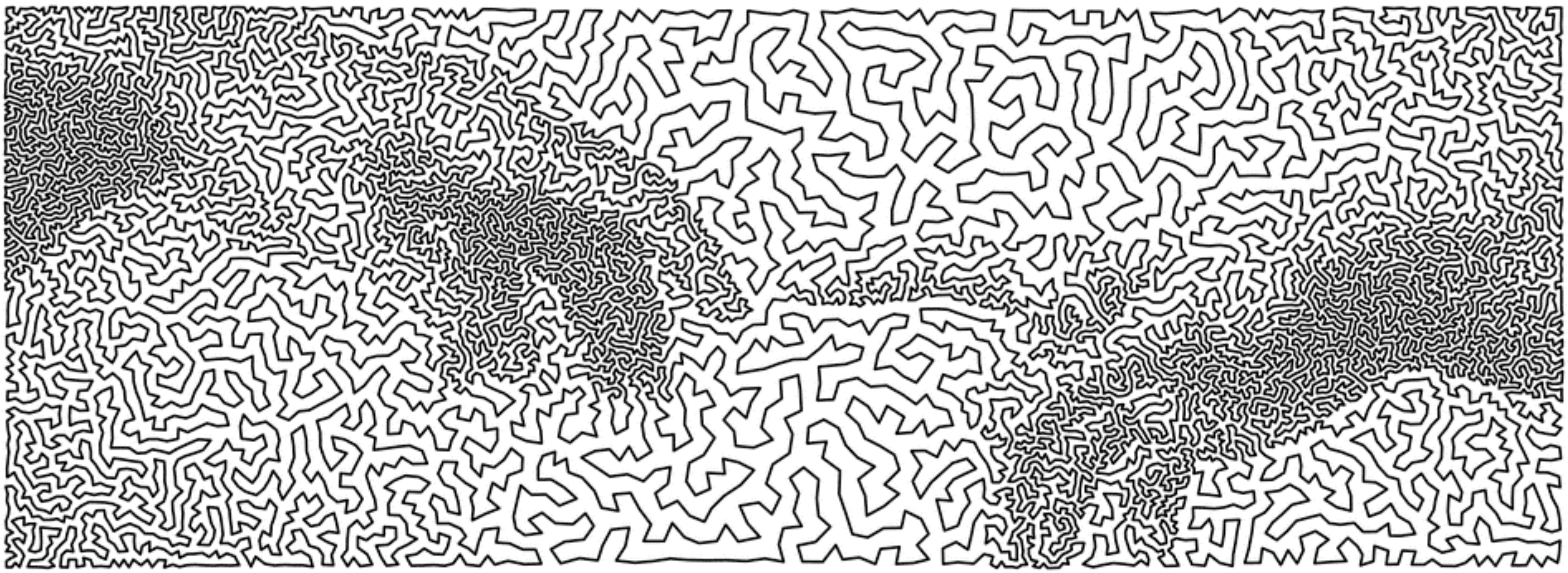


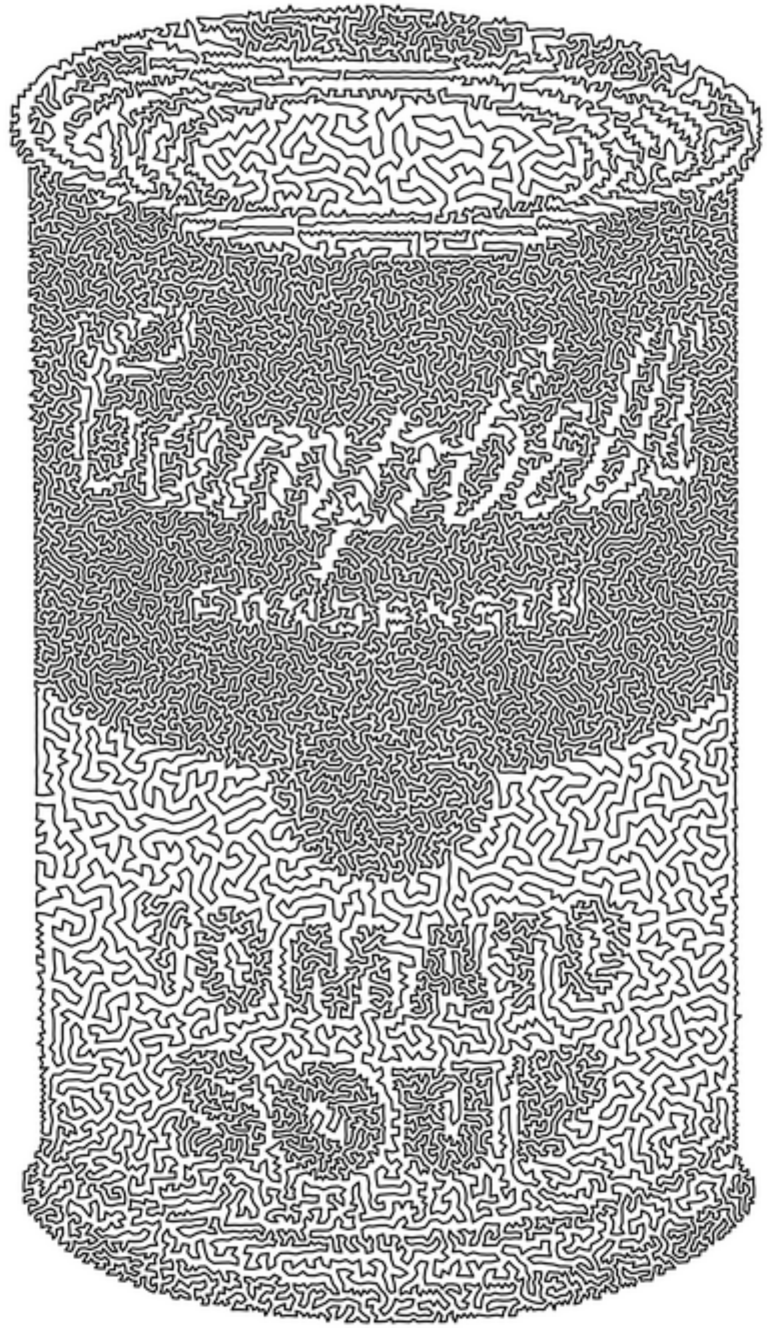


Hands

INFORMS 06

Pittsburgh, PA





What's Inside?

INFORMS 06

Pittsburgh PA

JMM 2007

San Diego CA

Bridges 2007

Donostia, Spain

This is!

INFORMS 06

Pittsburgh PA

JMM 2007

San Diego CA

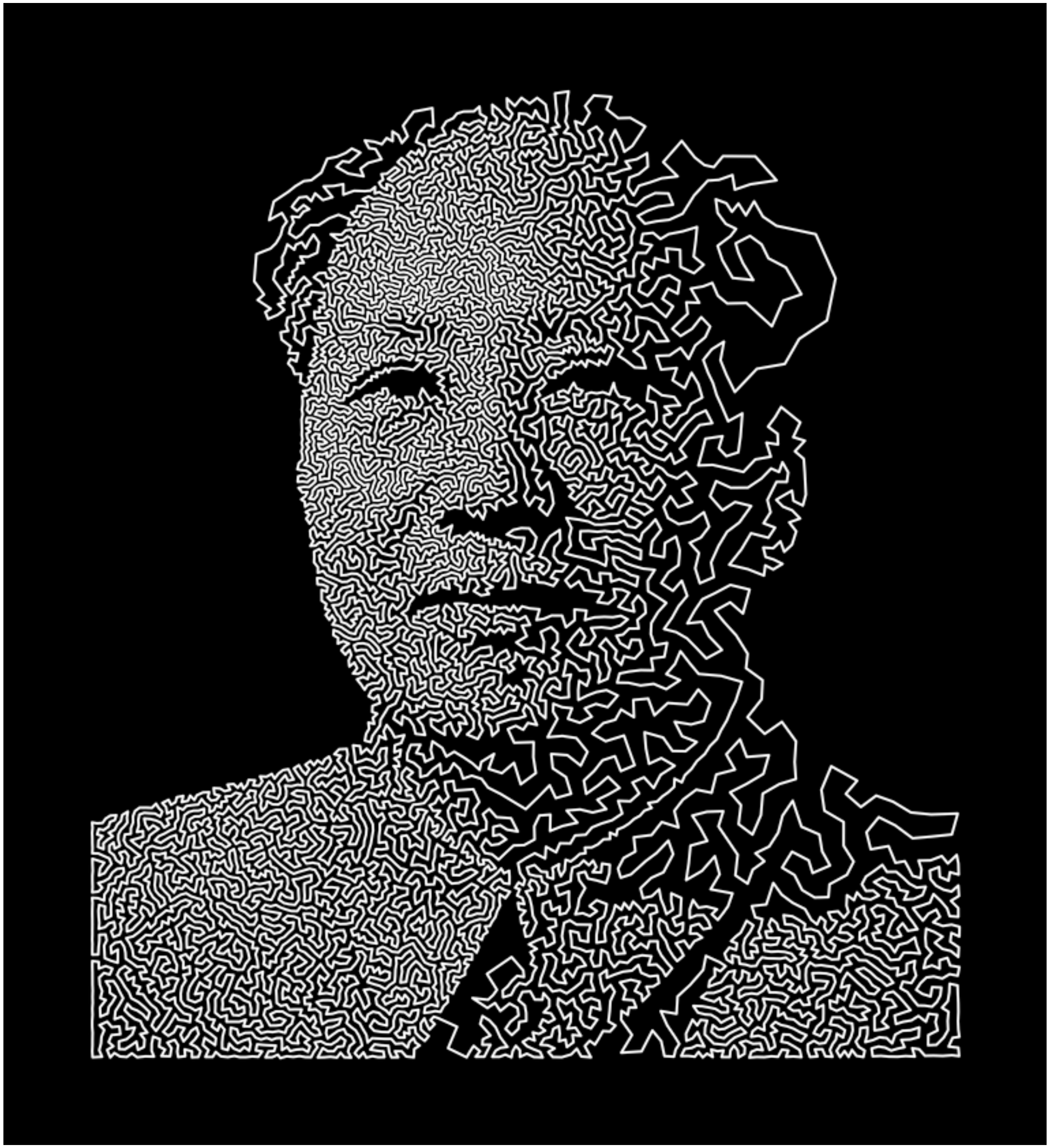
Bridges 2007

Donostia, Spain

One loop variation 1

INFORMS 06

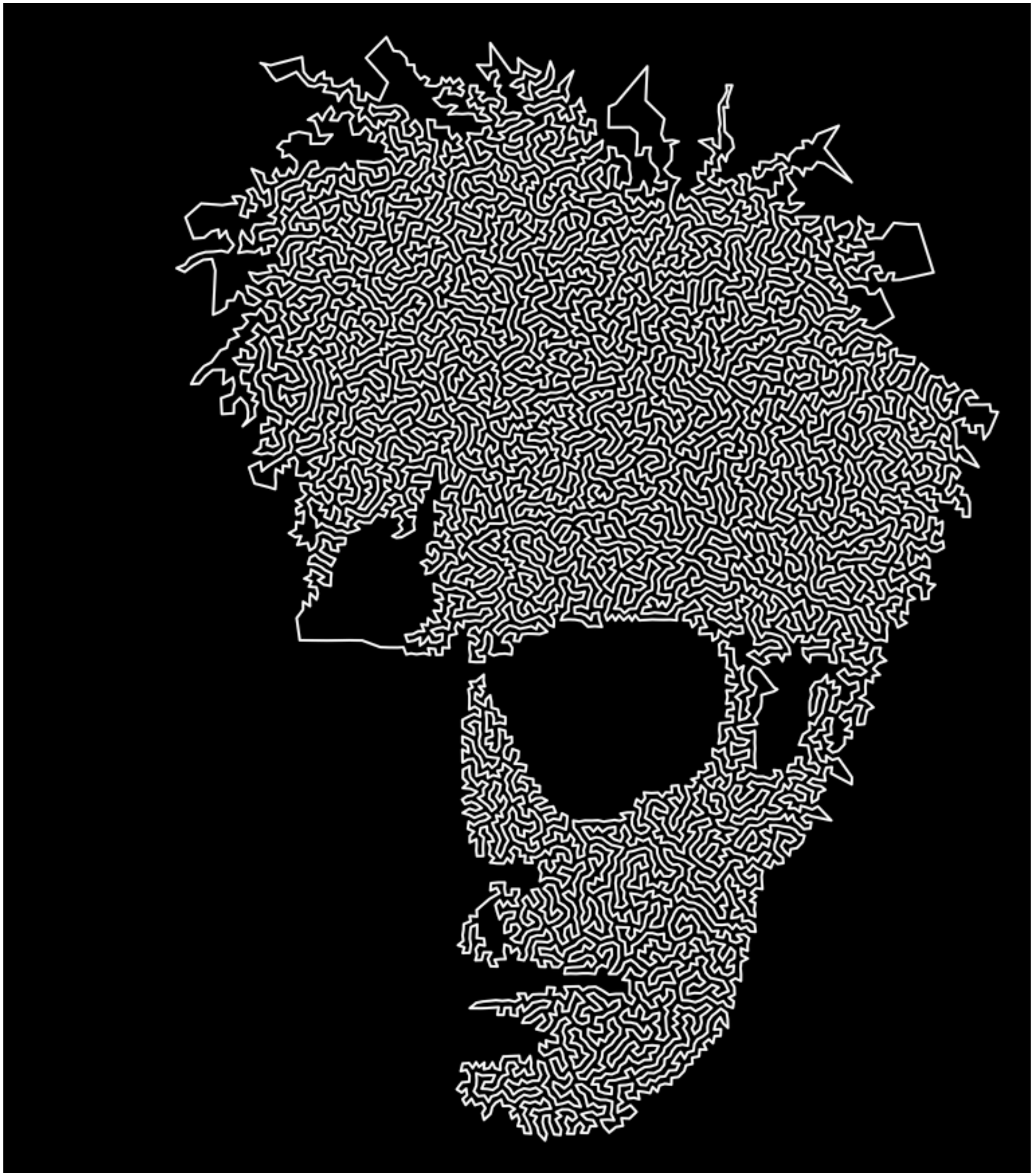
Pittsburgh PA



One loop variation 2

INFORMS 06

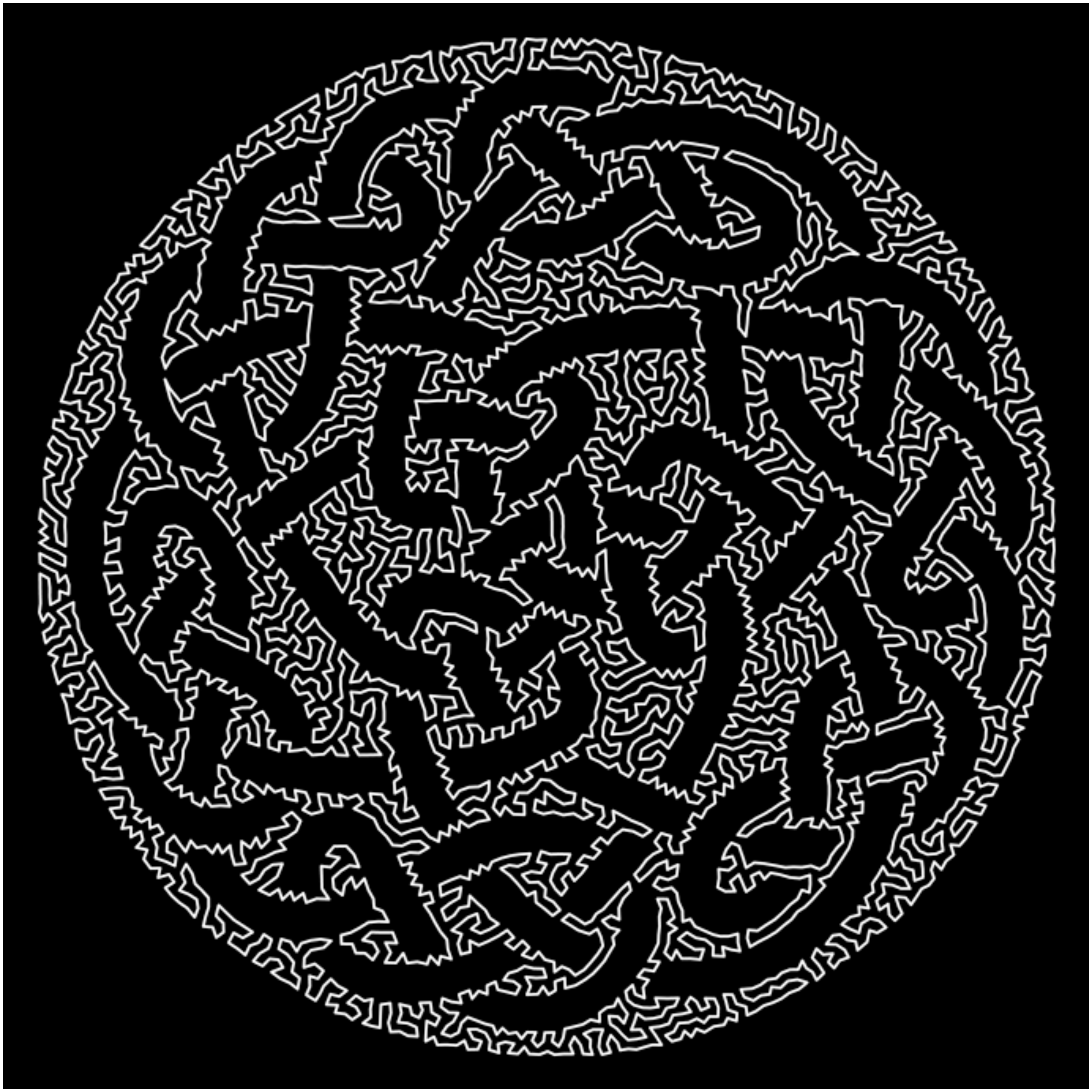
Pittsburgh PA



One loop variation 3

INFORMS 06

Pittsburgh PA



Knot?

CPAIOR06

Cork, Ireland

JMM 2007

New Orleans, LA

Bridges 2007

Donostia, Spain

One fish, two fish, red fish, black fish

JMM 2008

San Diego, CA

Bridges 2008

Leeuwarden, NL

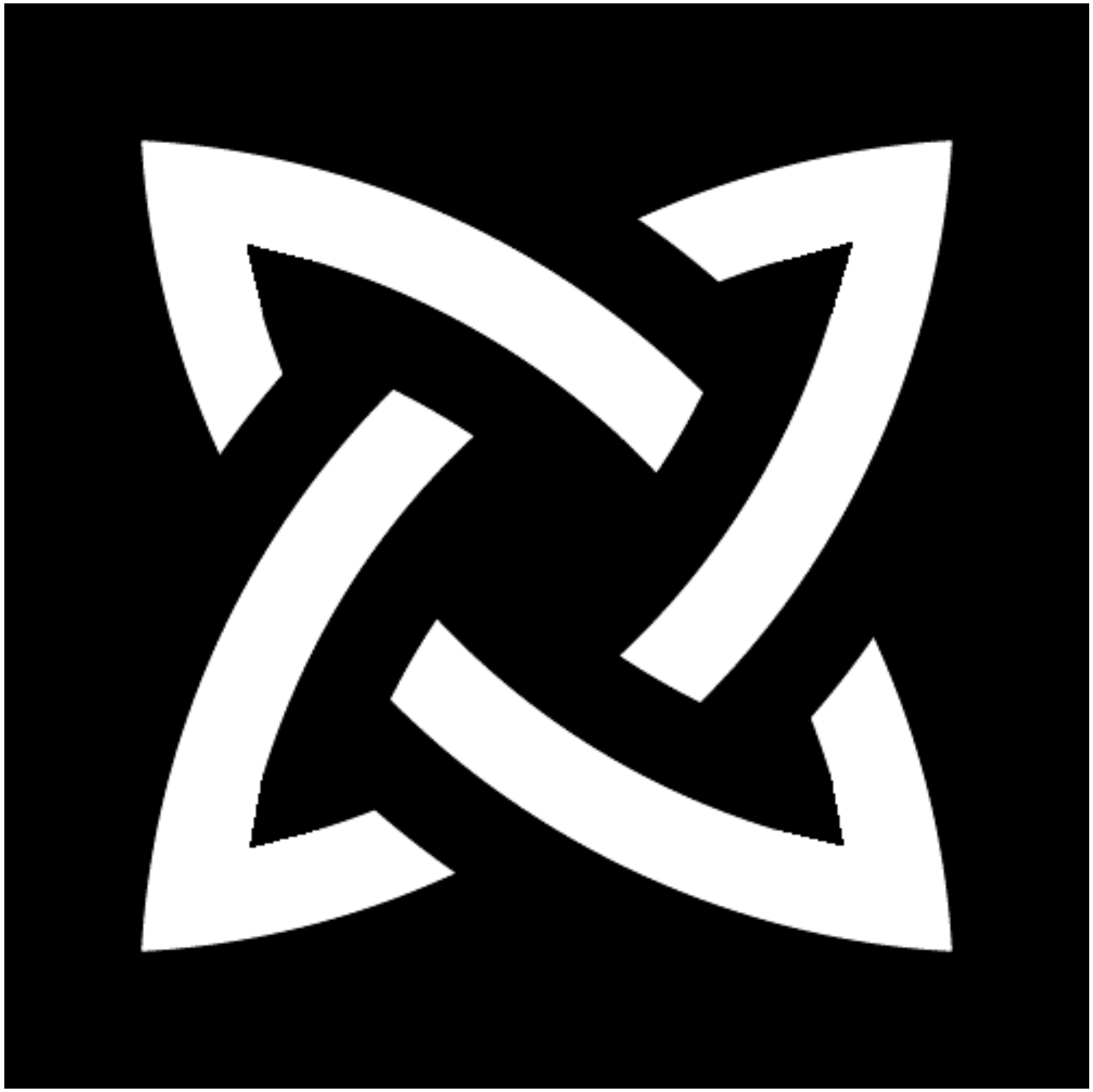
Outside Ring

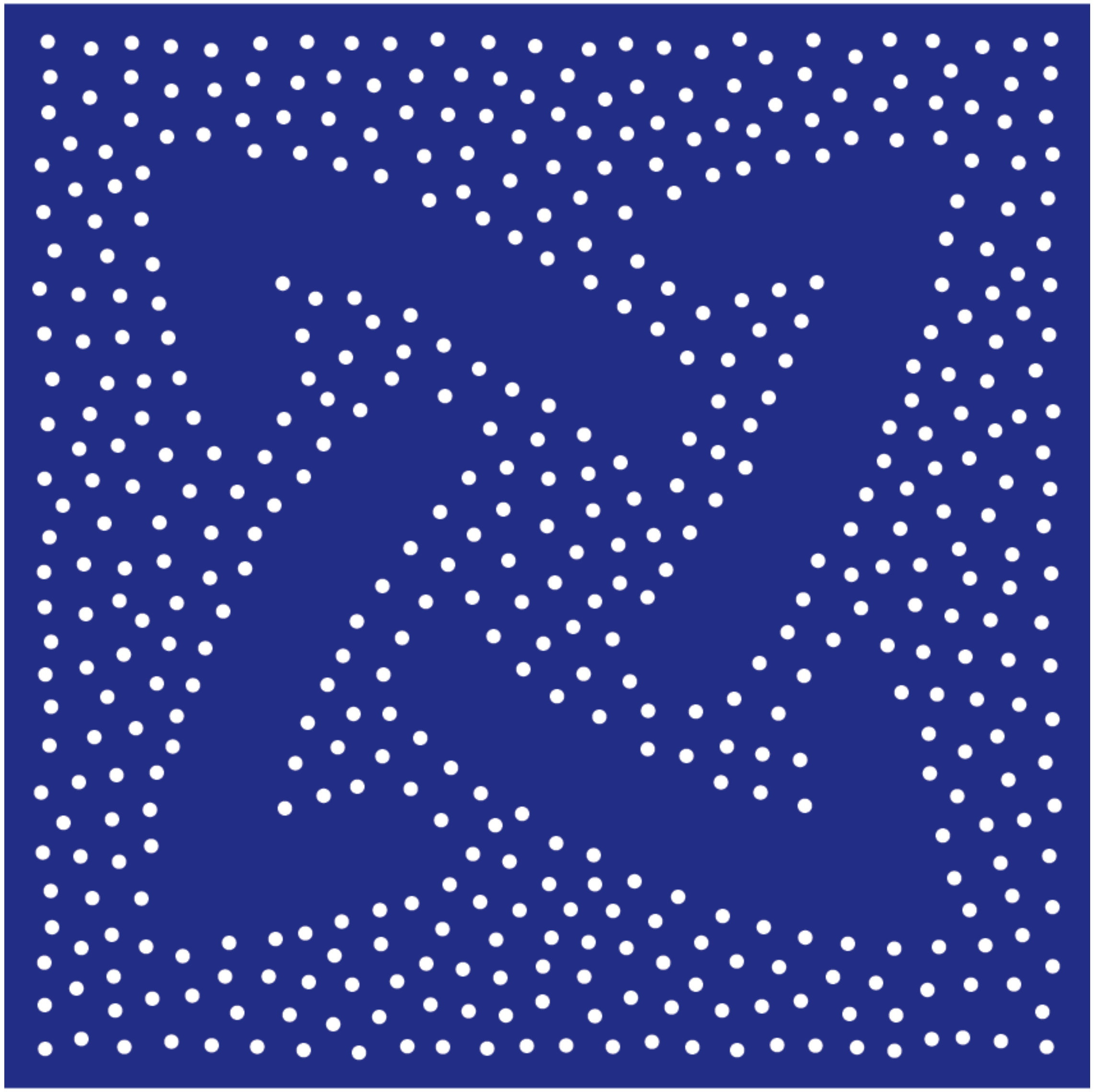
JMM 2008

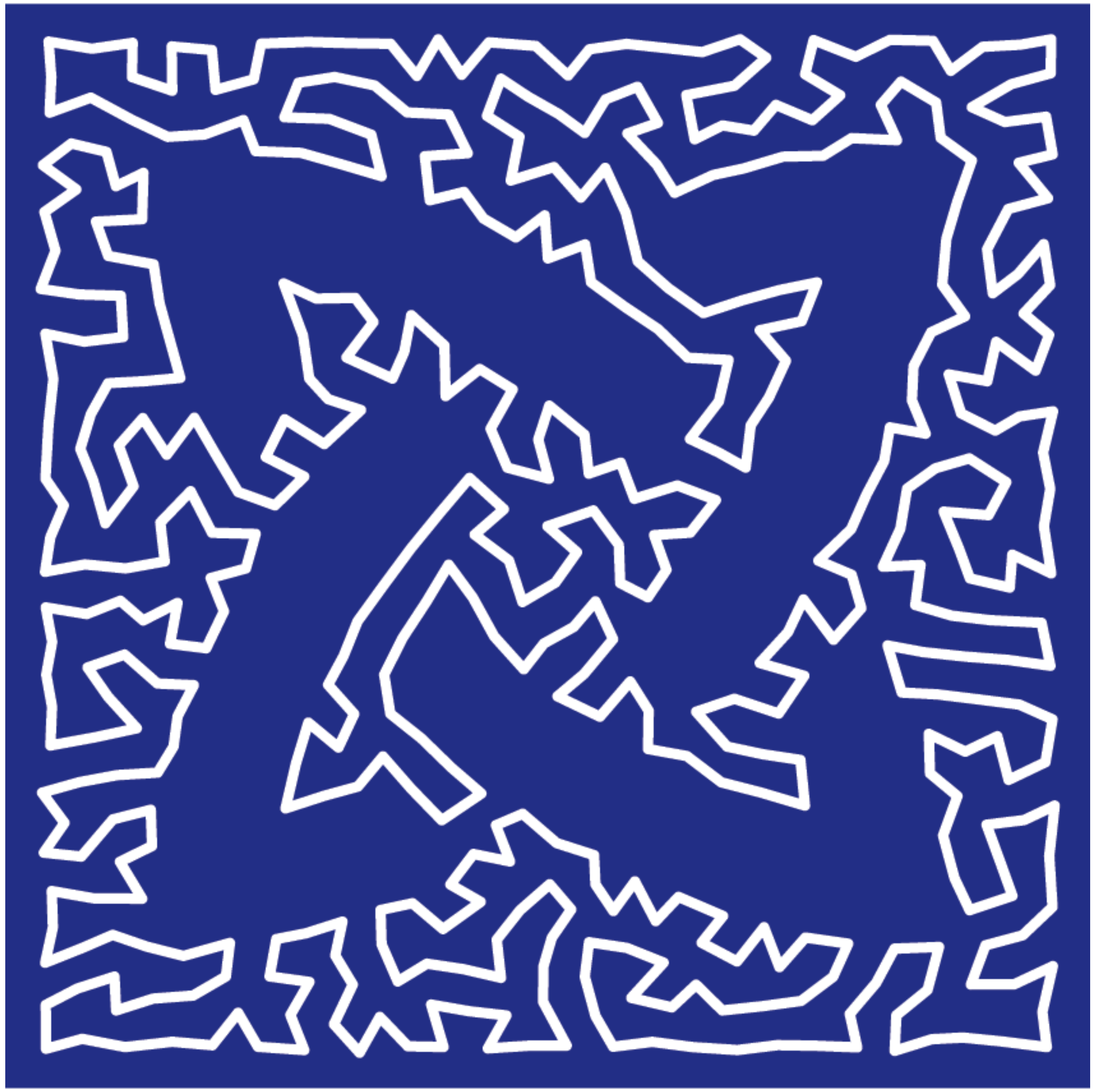
San Diego, CA

Bridges 2008

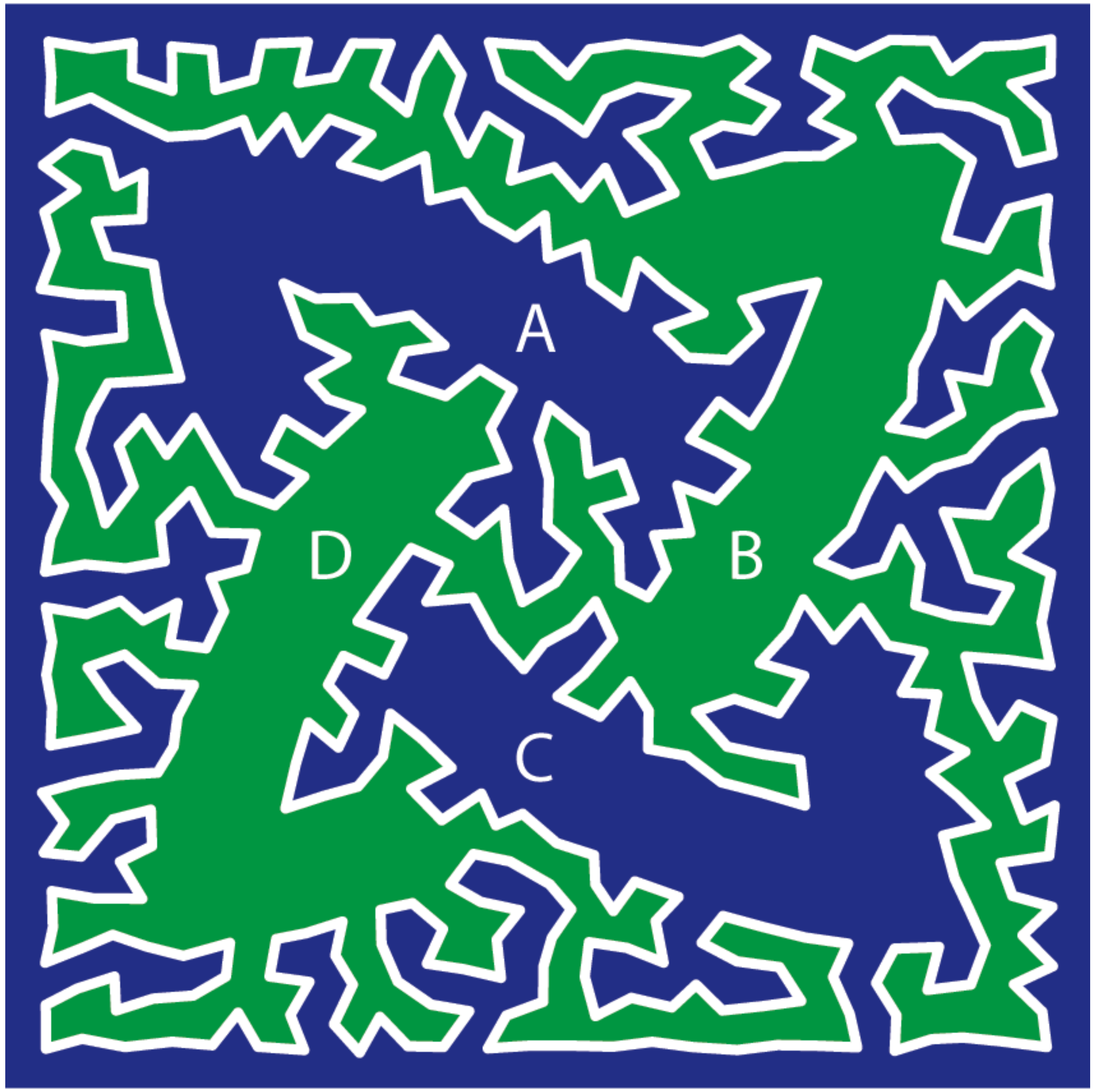
Leeuwarden, NL

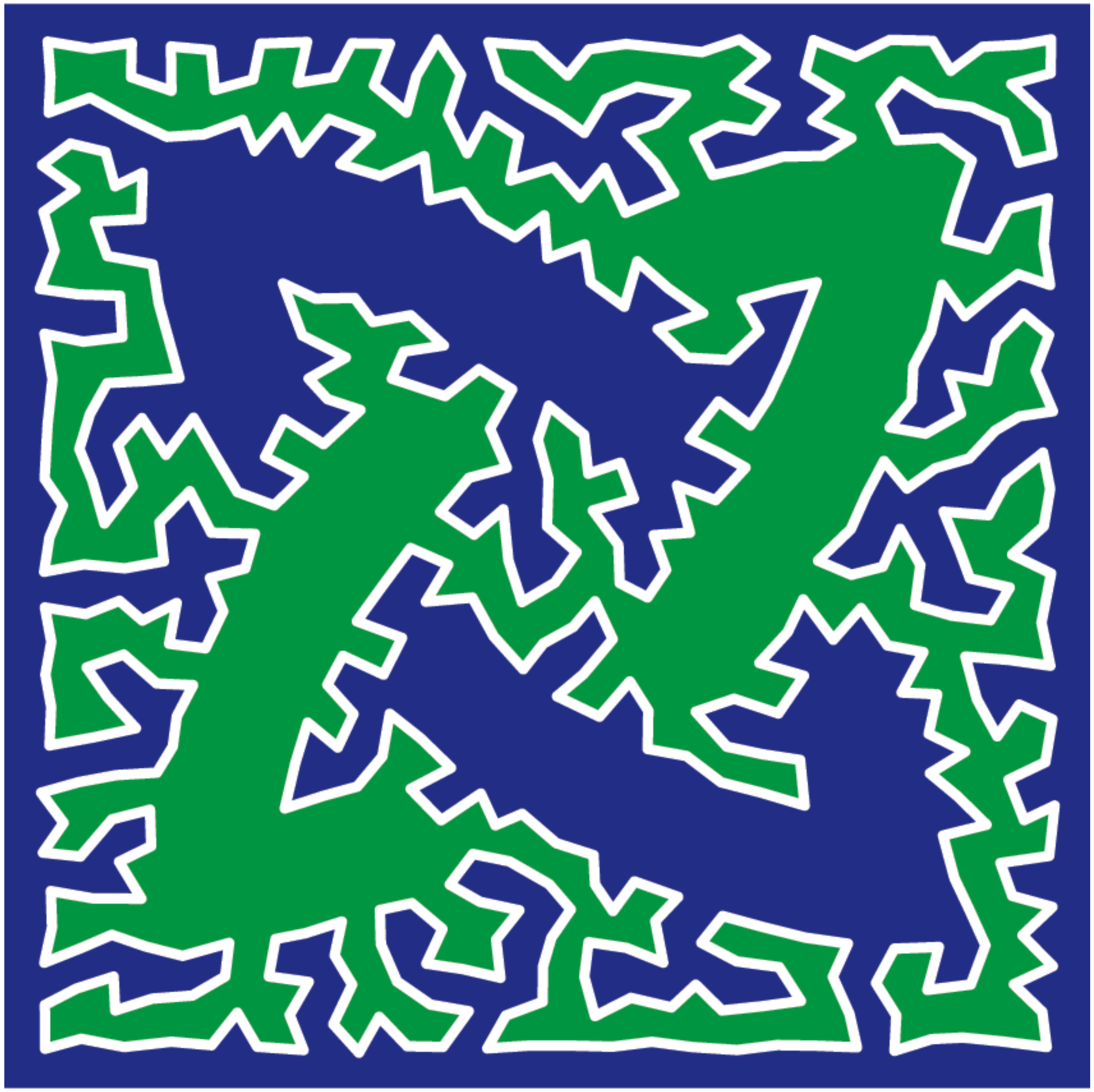






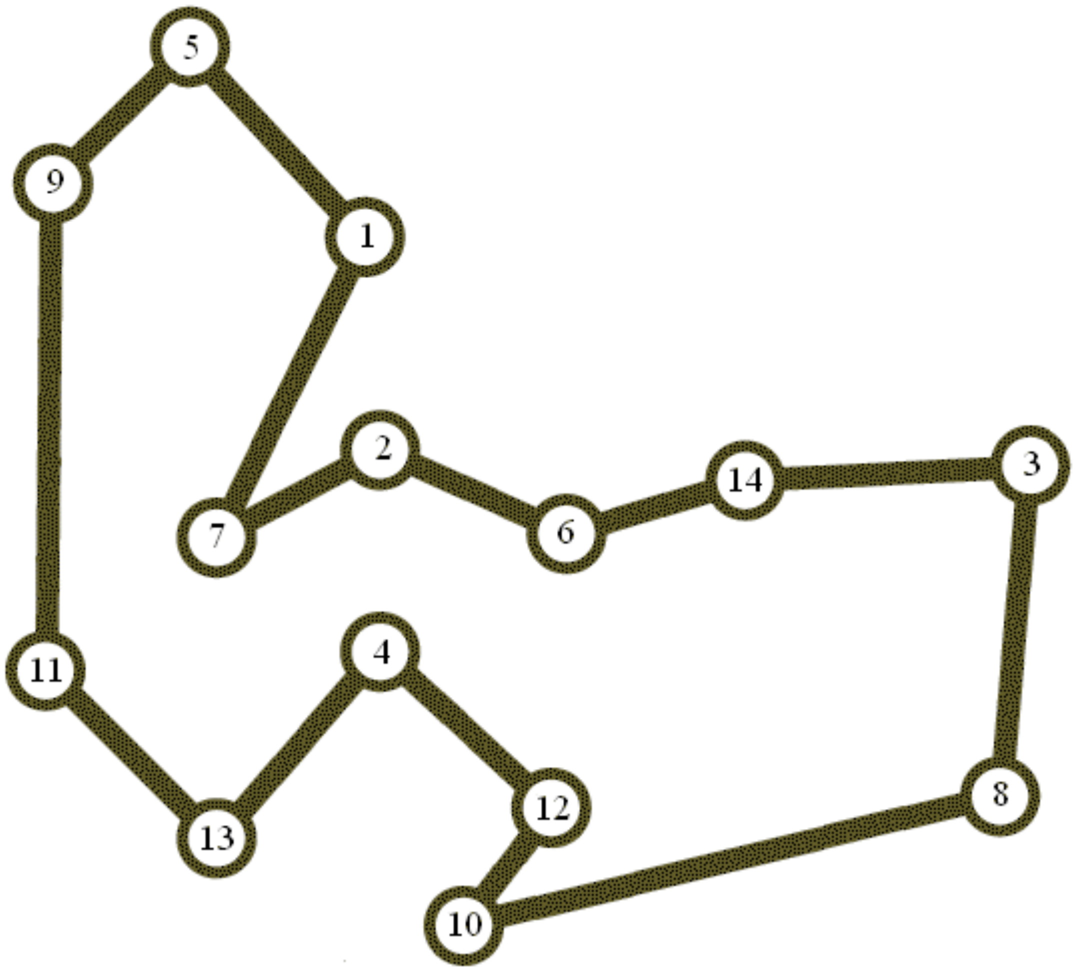

A
B
C
D


A
B
C
D



Solving TSPs with IP:

Eighth Judicial Circuit traveled by Lincoln in 1850.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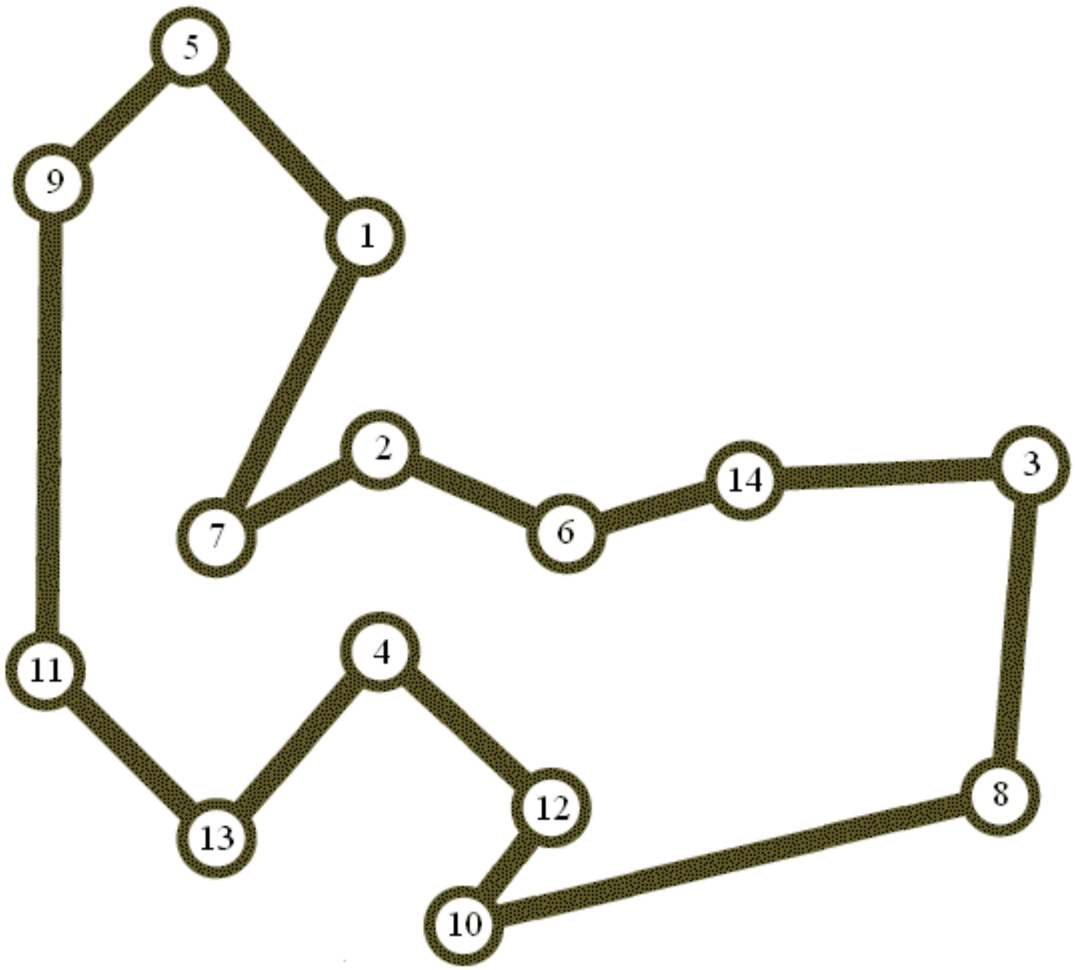
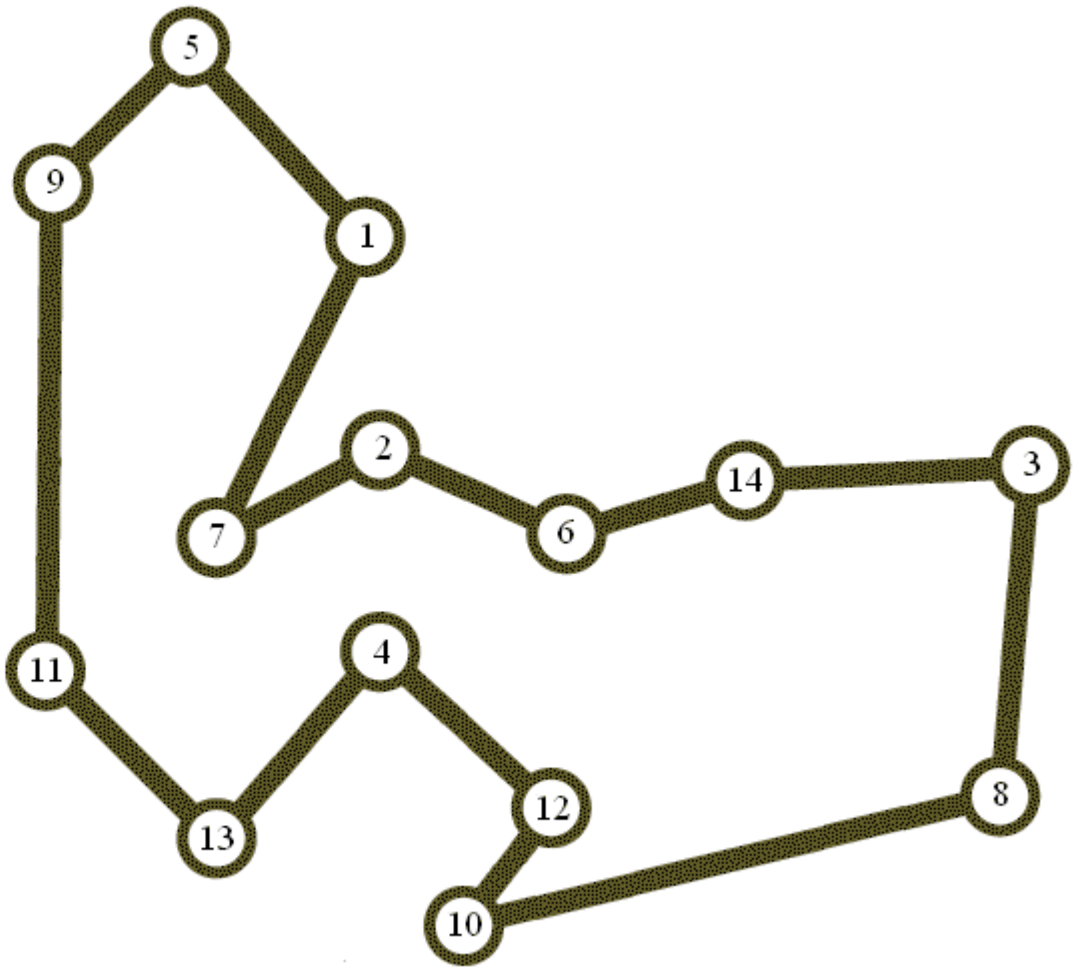
Variables: $x_e = 1$ if edge e is used in the tour



Variables: $x_e = 1$ if edge e is used in the tour



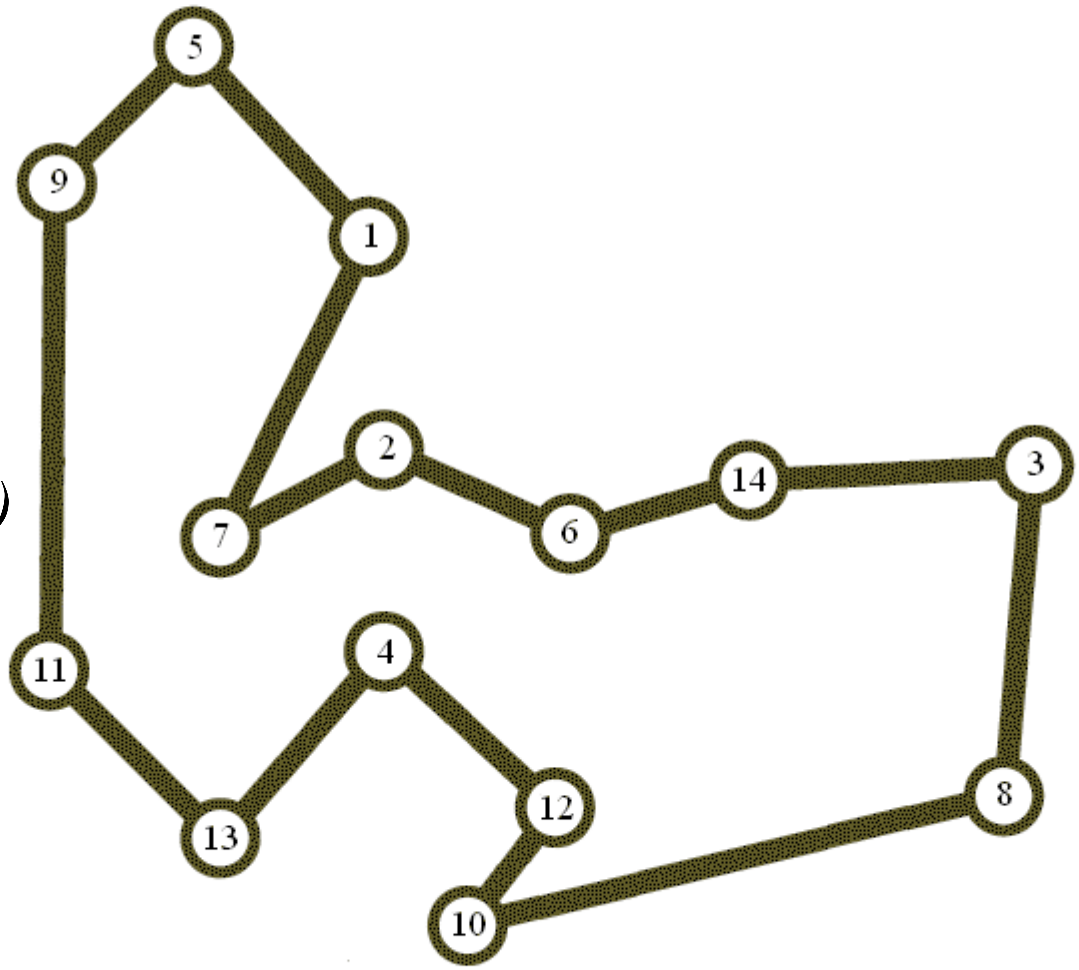
$x_{3,8} = 1$

(e = 3,8)

Variables: $x_e = 1$ if edge e is used in the tour

$x_{9,11} = 1$

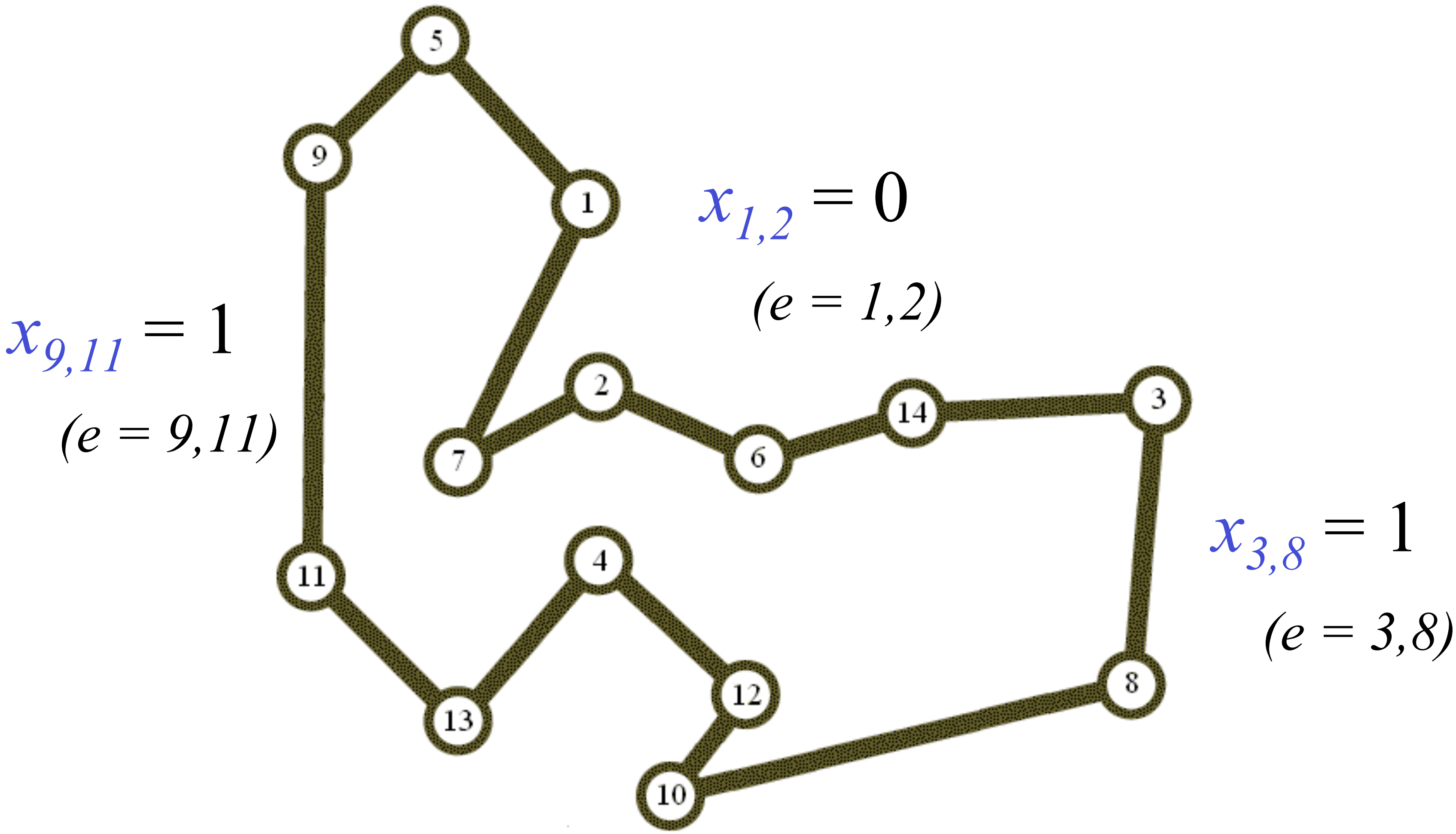
(e = 9,11)



$x_{3,8} = 1$

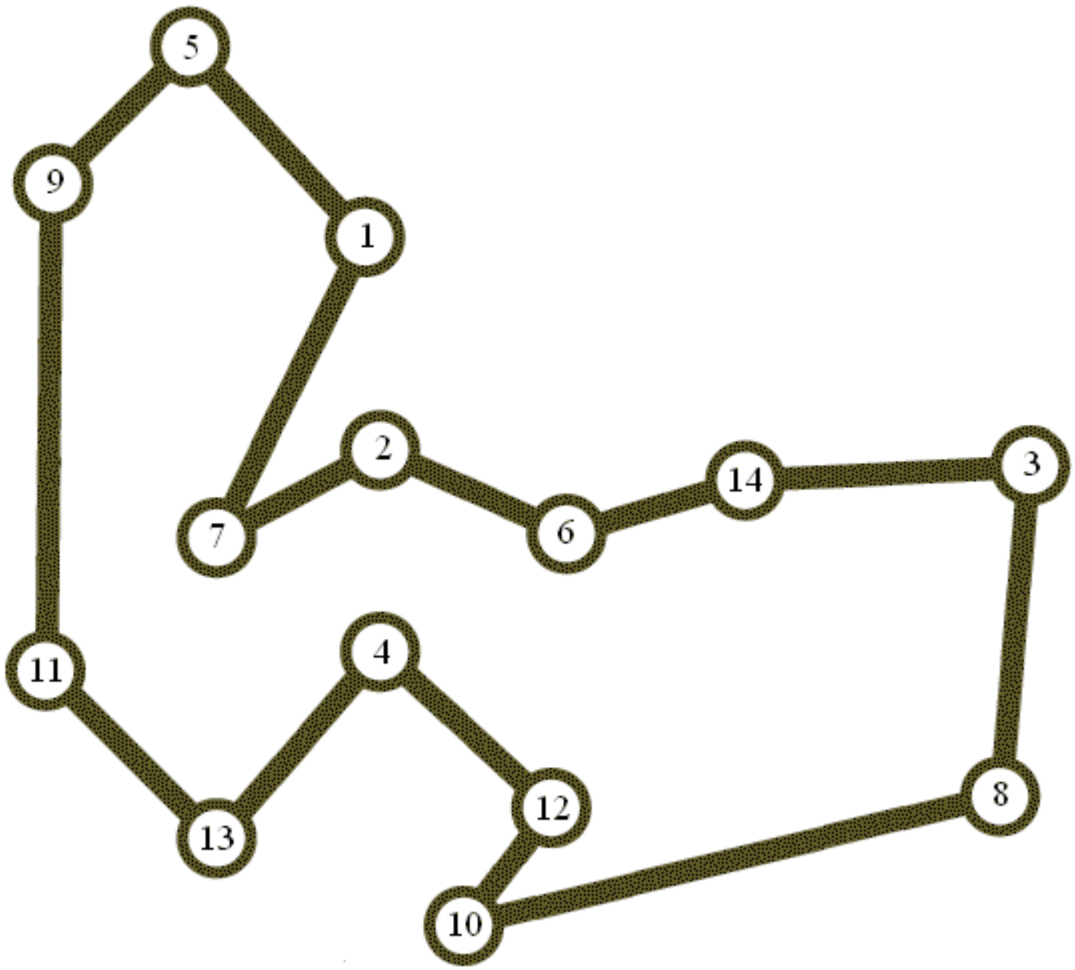
(e = 3,8)

Variables: $x_e = 1$ if edge e is used in the tour



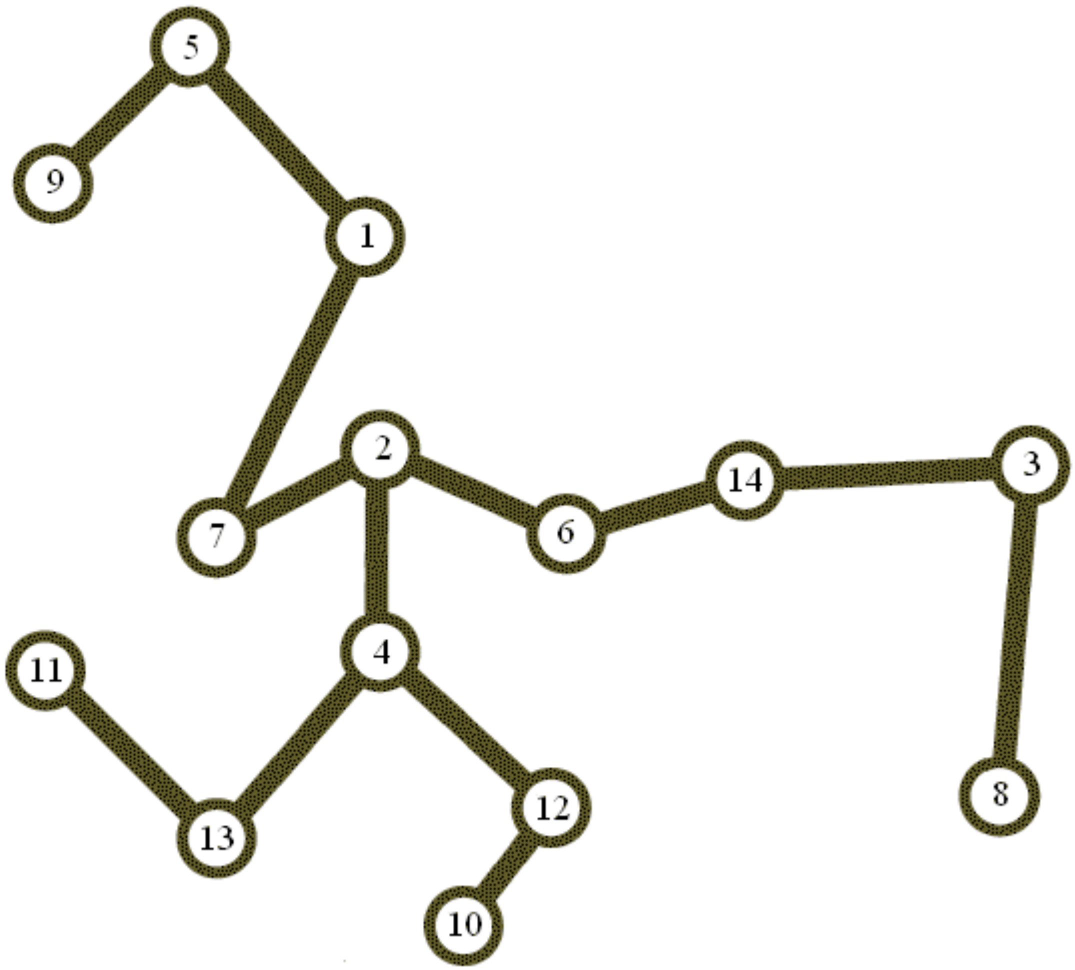
Objective: minimize length of tour

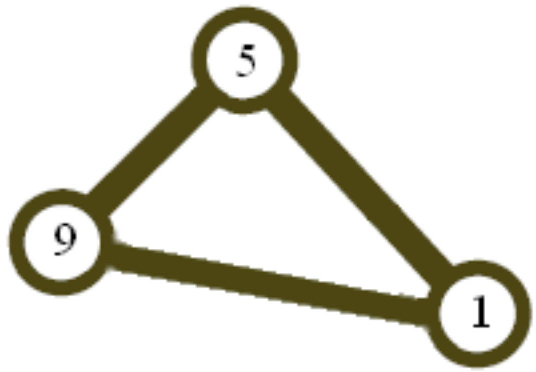
total cost $= \Sigma\ (c_e\, x_e$: all edges e)



Constraints: each city v must be touched twice

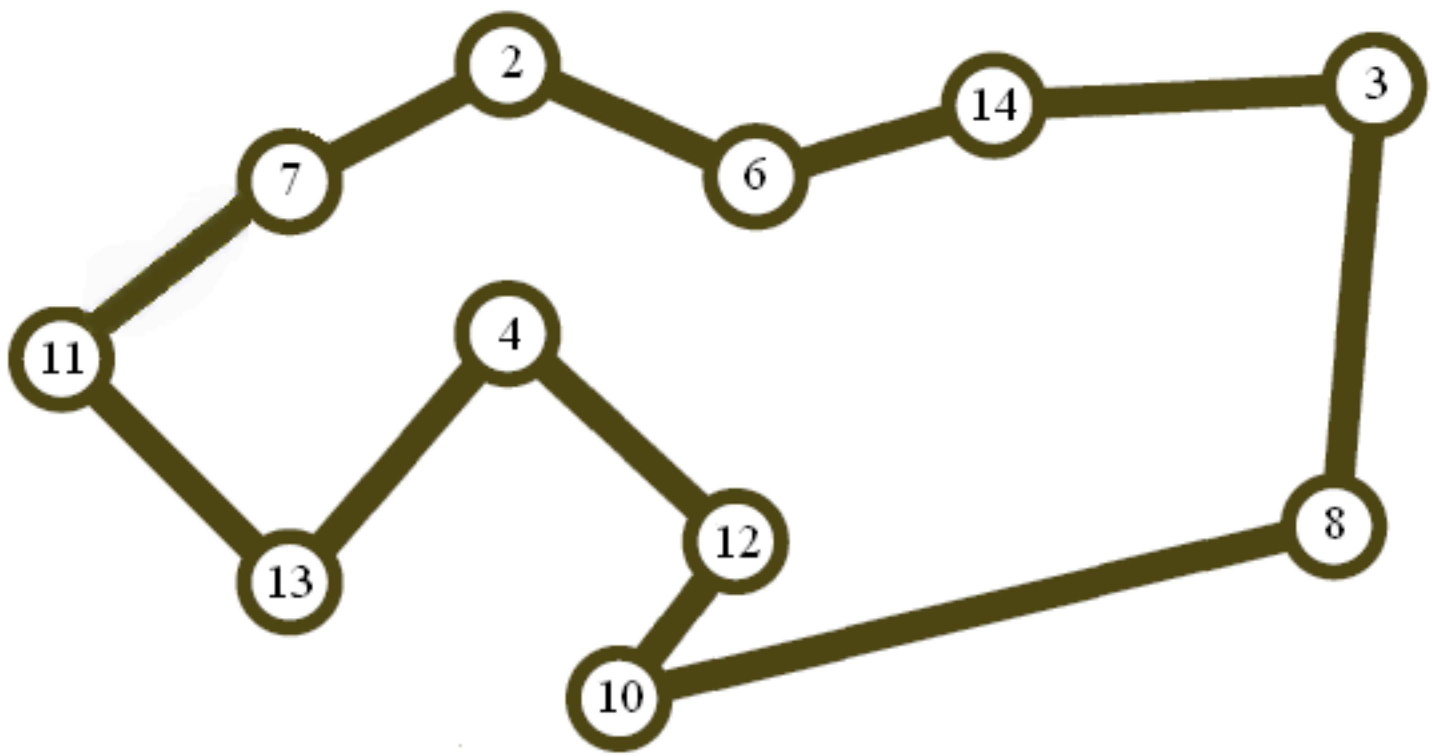
$$\Sigma\,(\,x_e : \text{all edges } e \text{ that touch } v\,) = 2$$





Constraints: no subtours!

$$\Sigma \; (\, x_e : \text{all } e \text{ with both endpoints in } S \,) < |S|$$



Solving TSPs with IP (DFJ formulation):

Variables: $x_e = 1$ if edge e is used in the tour

Objective: minimize length of tour

total cost $= \Sigma\ (c_e\, x_e : \text{all edges } e)$

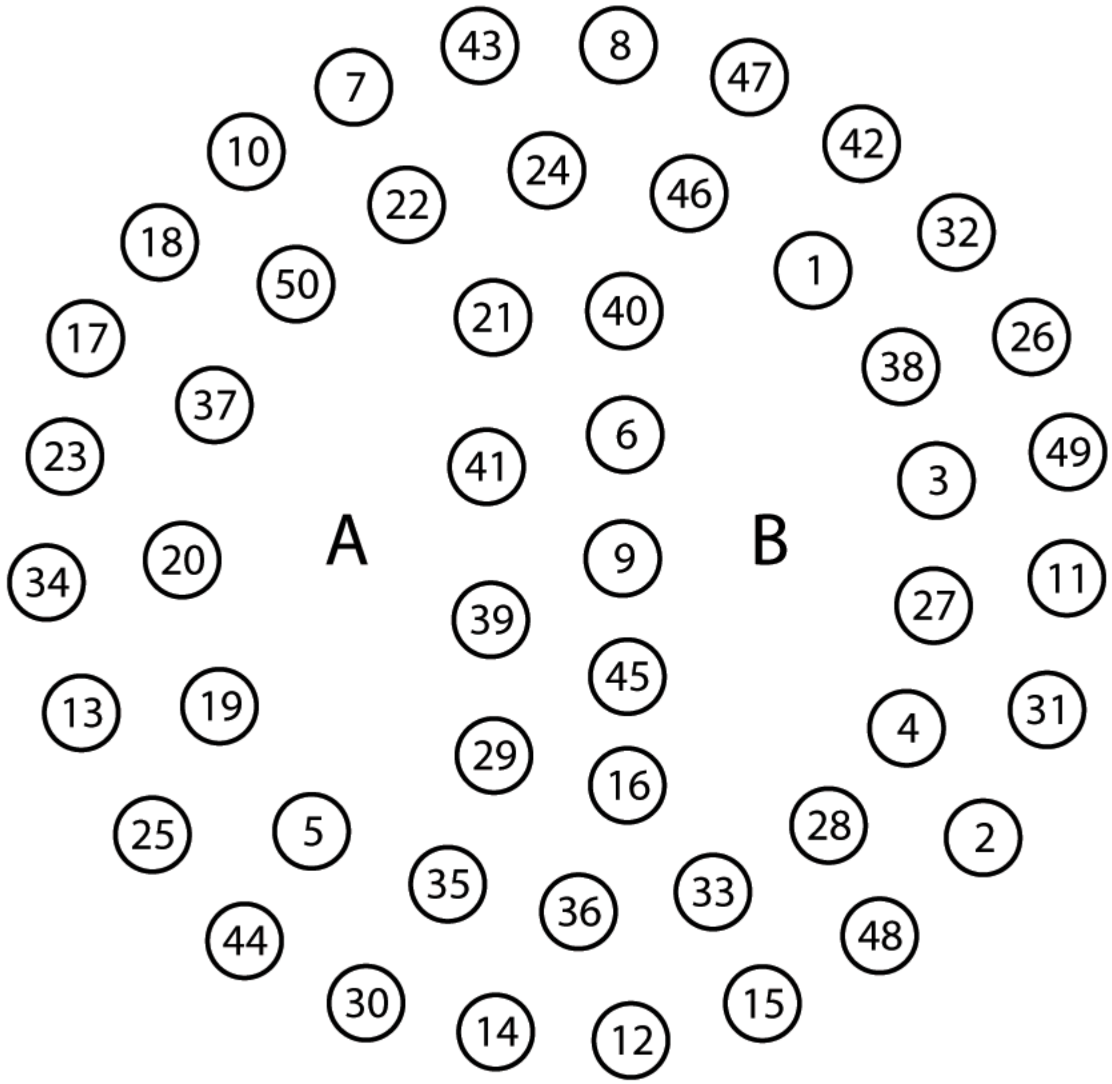
Constraints: each city v must be touched twice

$\Sigma\ (x_e : \text{all edges } e \text{ that touch } v) = 2 \qquad \forall\ v$

Constraints: no subtours are allowed

$\Sigma\ (x_e : \text{all } e \text{ w/both endpts in } S) < |S| \qquad \forall\ S$

An example: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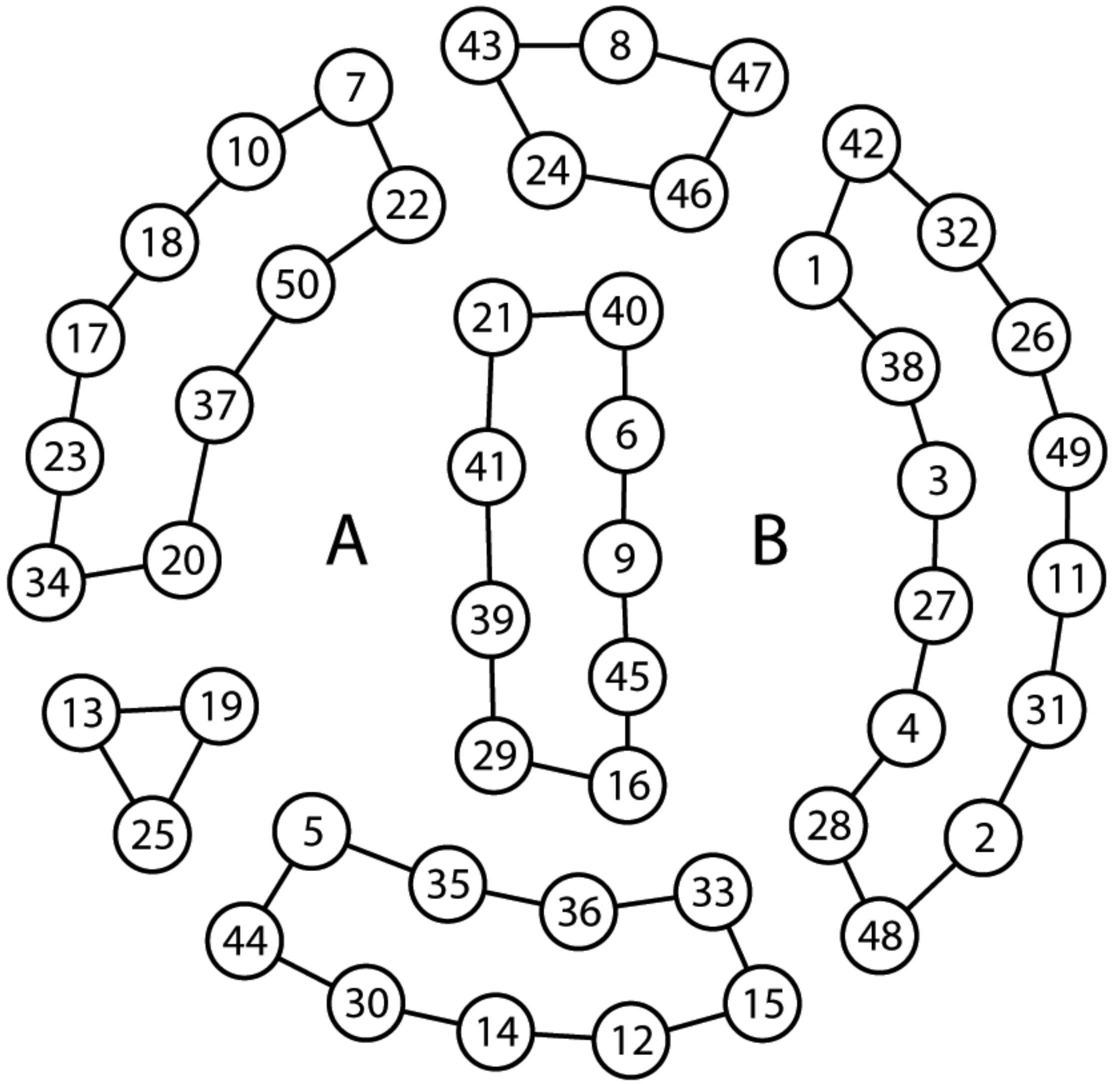
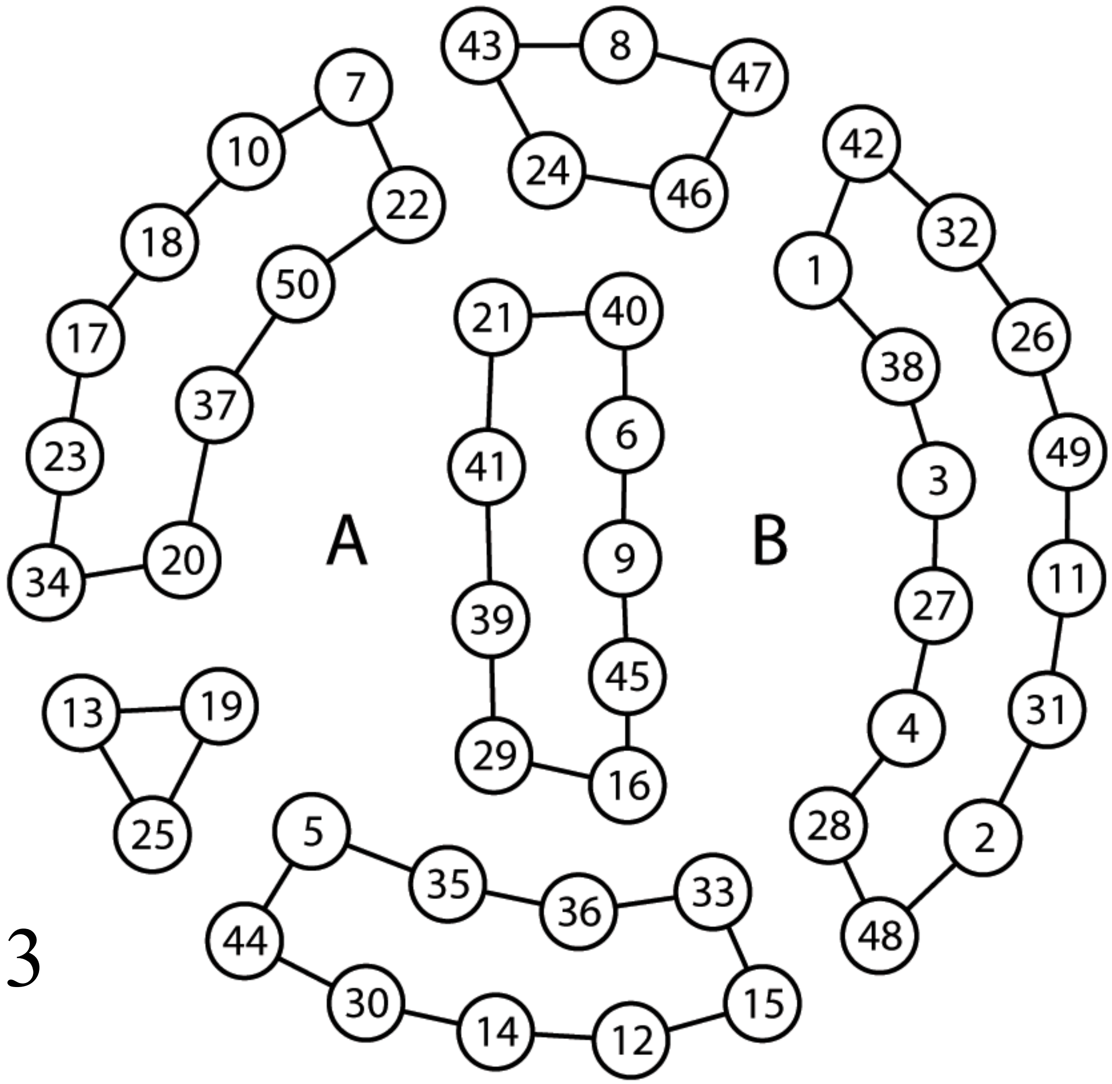
Stage 1: 6 subtours.

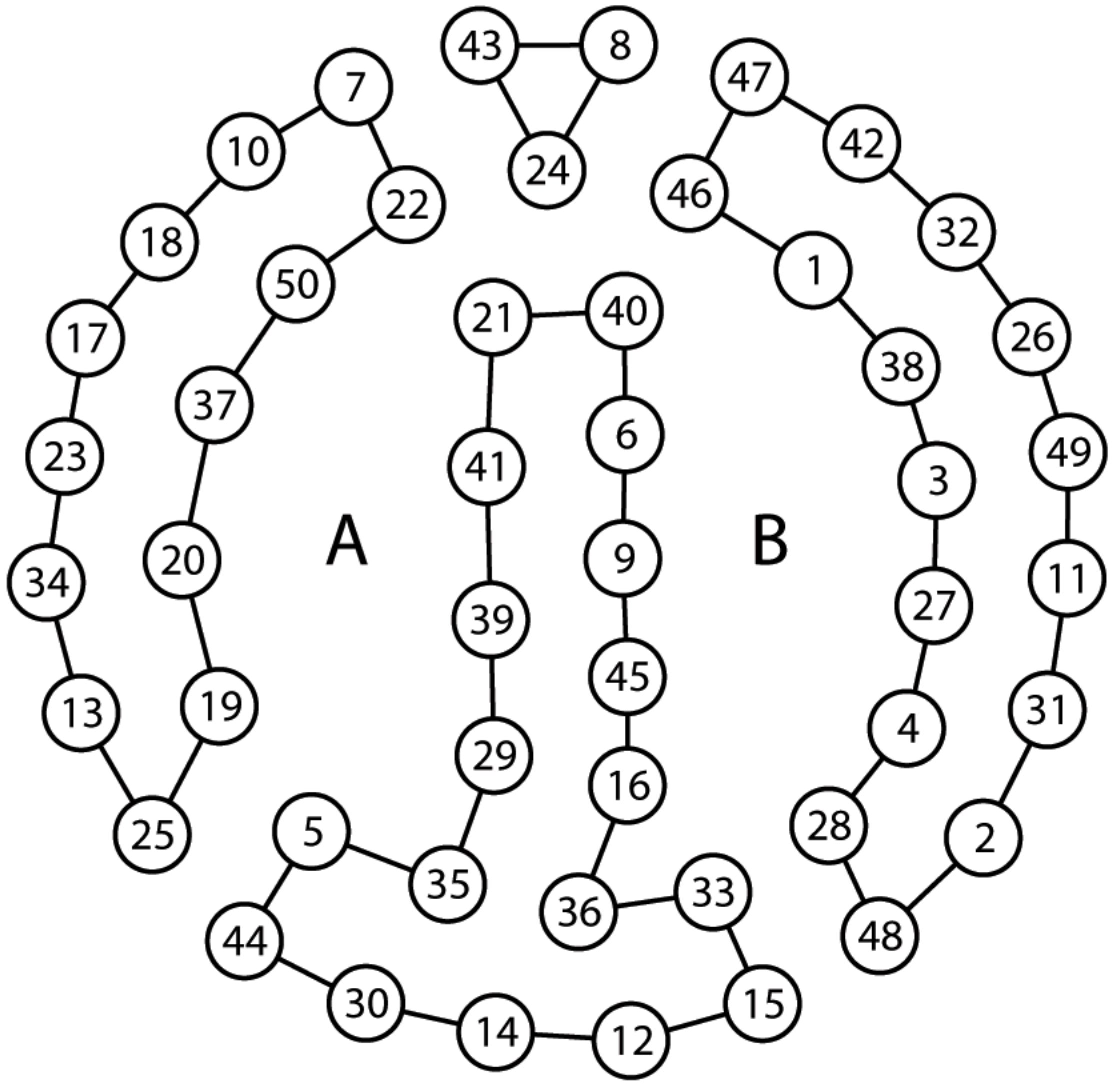


Stage 1: 6 subtours.

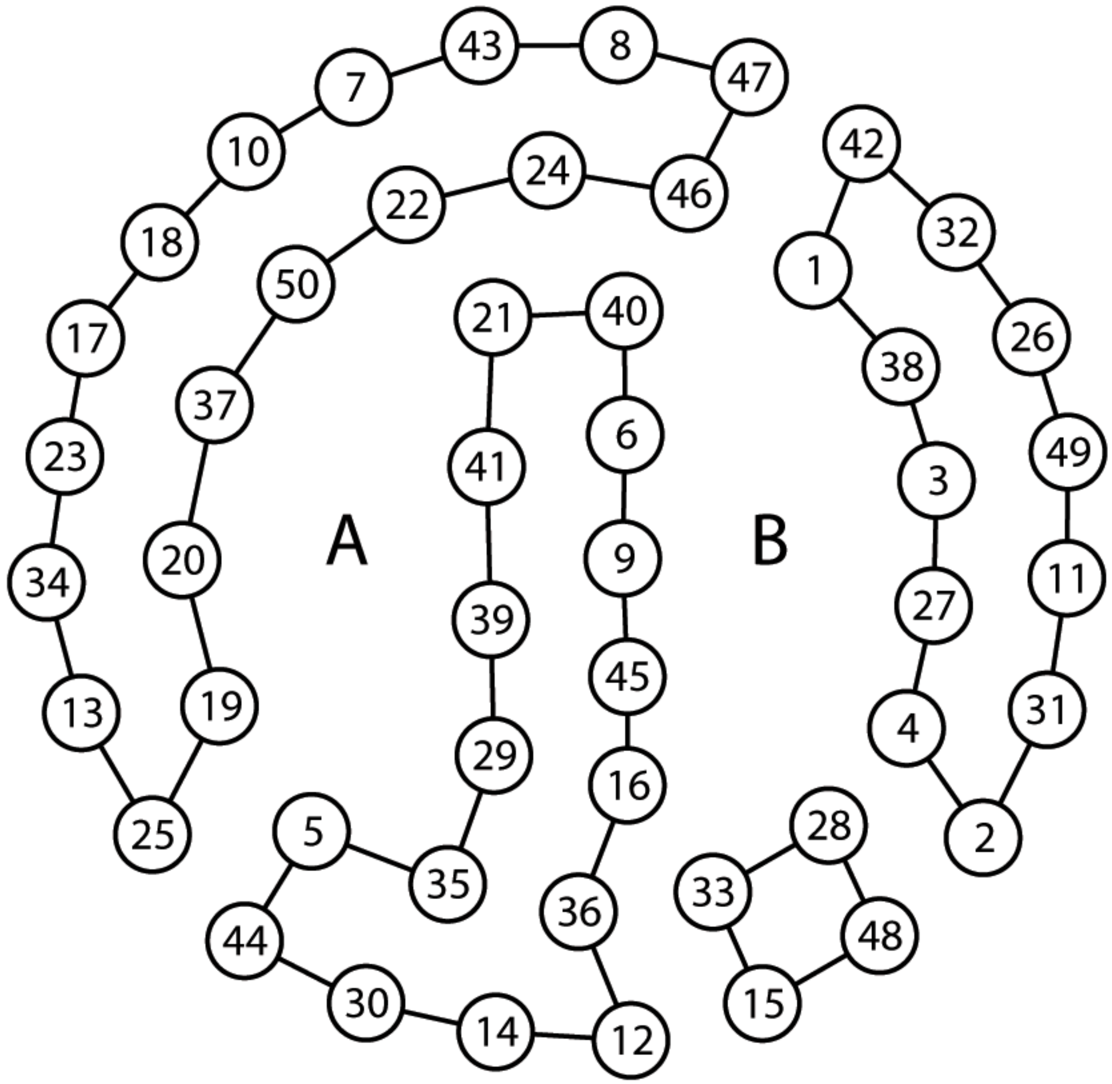


$$x_{13,19} + x_{13,25} + x_{19,25} < 3$$

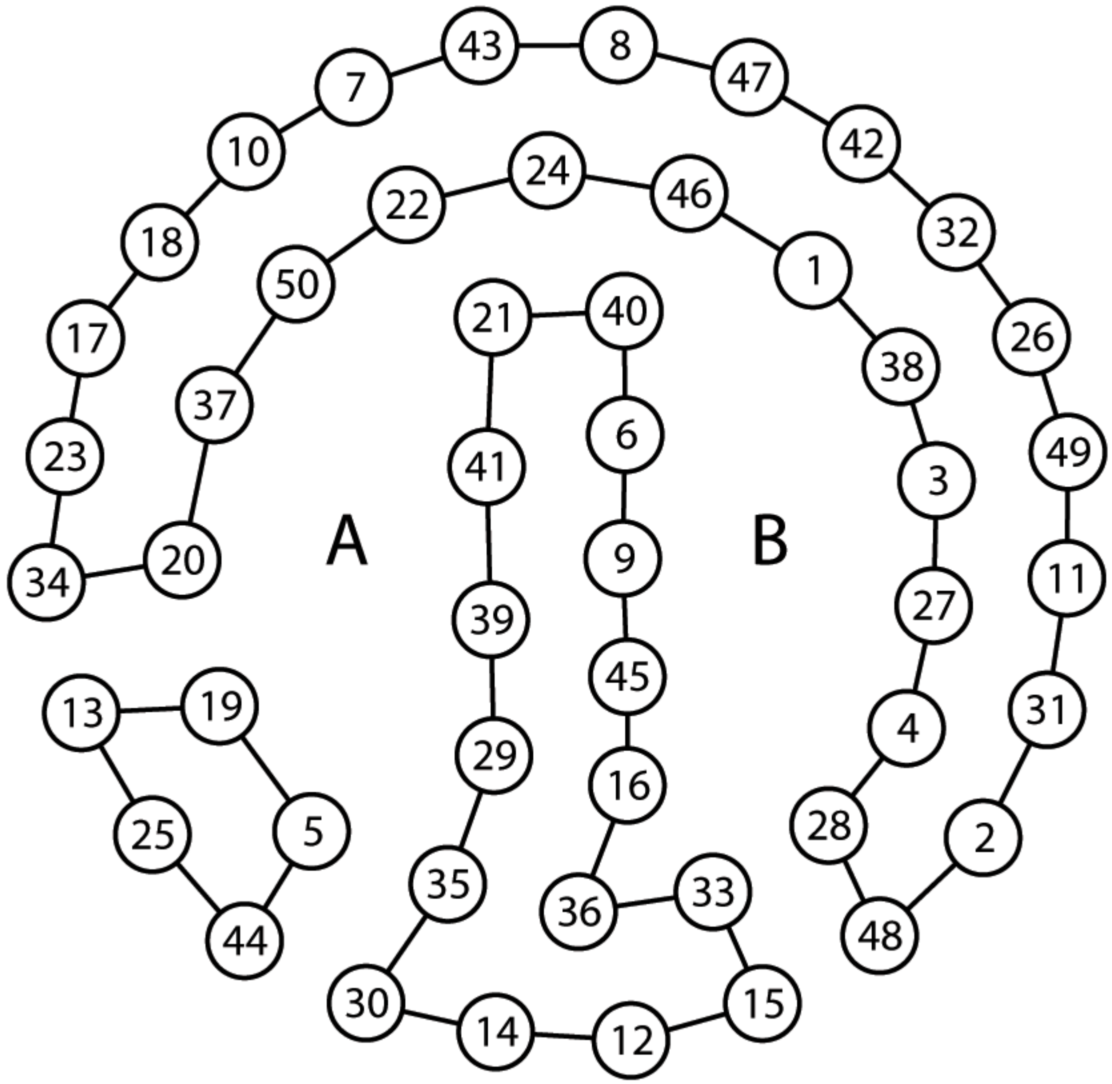
Stage 2: 4 more subtours.



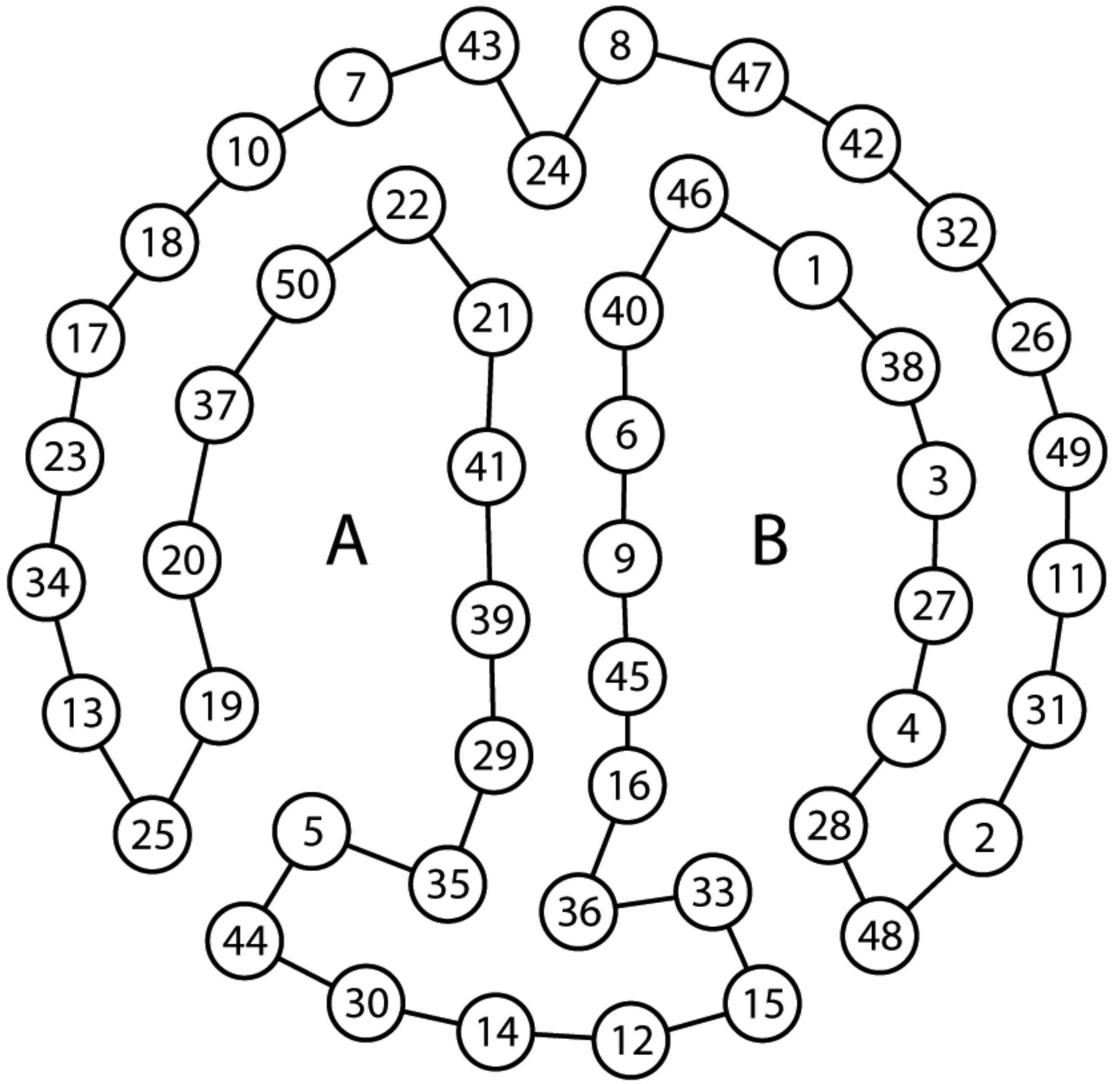
Stage 3: 4 more subtours.

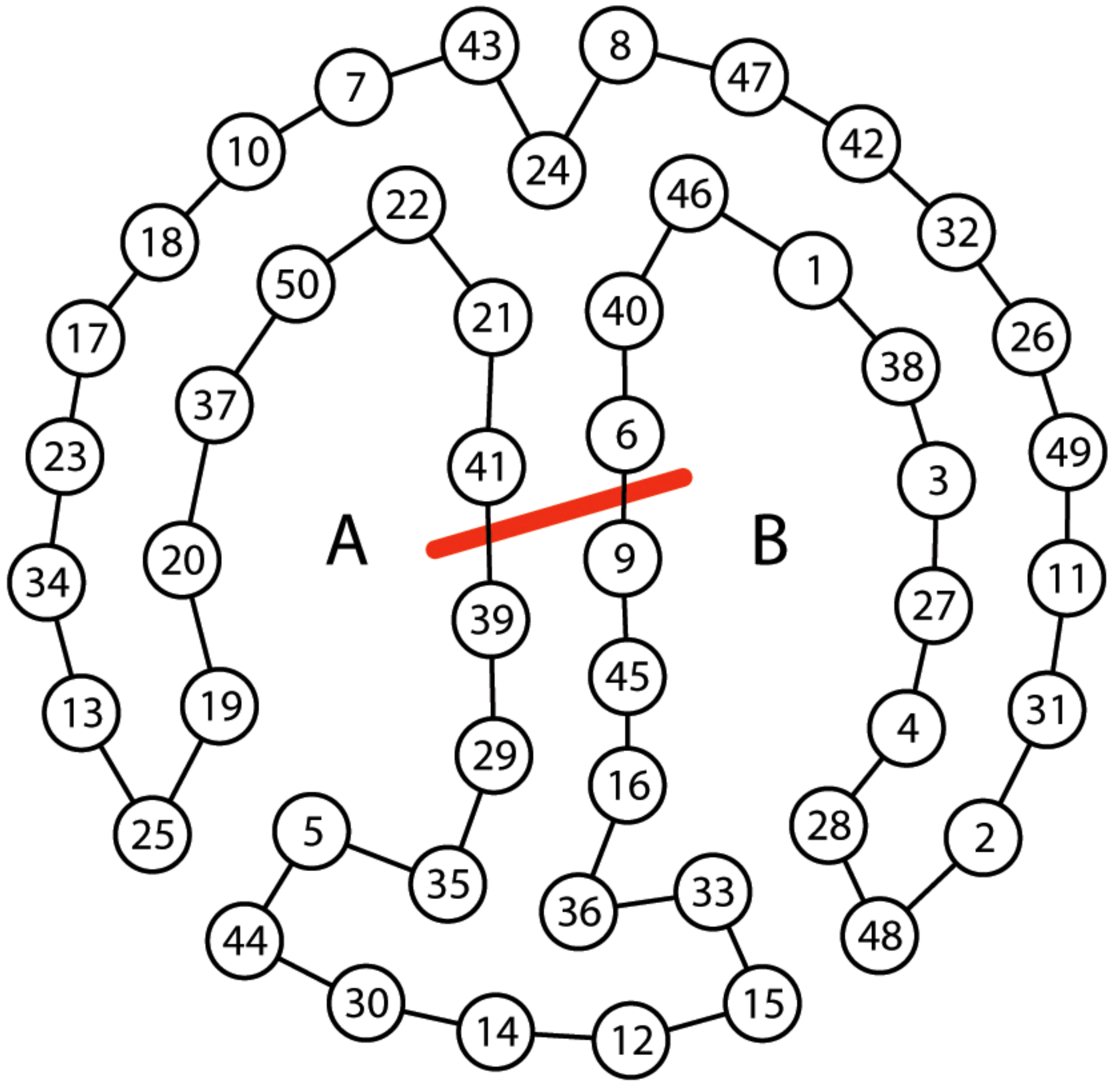


Stage 4: 3 more subtours.



Stage 5: success!

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Definition:

$$\chi(l_{A,B}) = \{ \text{ all edges } e \text{ that cross line } l_{A,B} \}$$

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Note:

A and B lie on the same side of the tour

$$\Longleftrightarrow$$

$$\Sigma\,(x_e : \text{all edges } e \text{ in } \chi(l_{A,B})\ ) \text{ is even}$$

$$\Longleftrightarrow$$

$$\Sigma\,(x_e : \text{all edges } e \text{ in } \chi(l_{A,B})\ ) = 2\, y_{A,B}$$

Definition:

$$\chi(l_{A,B}) = \{ \text{ all edges } e \text{ that cross line } l_{A,B} \}$$

Equivalently:

A and B lie on opposite sides of the tour

$$\Longleftrightarrow$$

$$\Sigma\, (x_e : \text{all edges } e \text{ in } \chi(l_{A,B})\,) \text{ is odd}$$

$$\Longleftrightarrow$$

$$\Sigma\, (x_e : \text{all edges } e \text{ in } \chi(l_{A,B})\,) = 2\, y_{A,B} + 1$$

The 'Big' Idea:

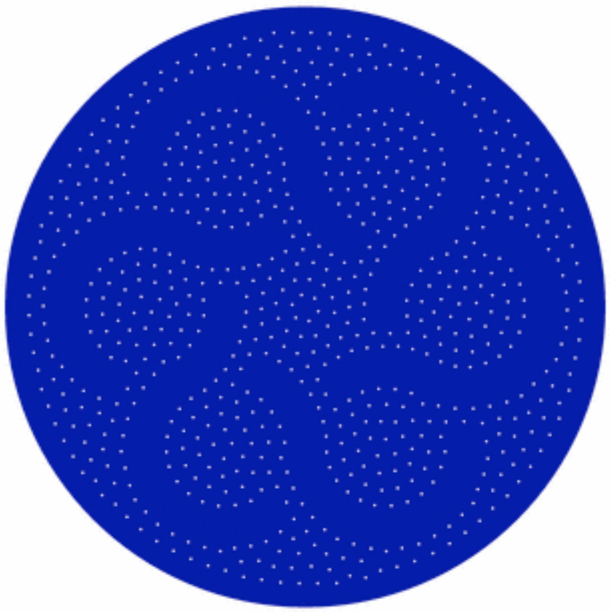
Use

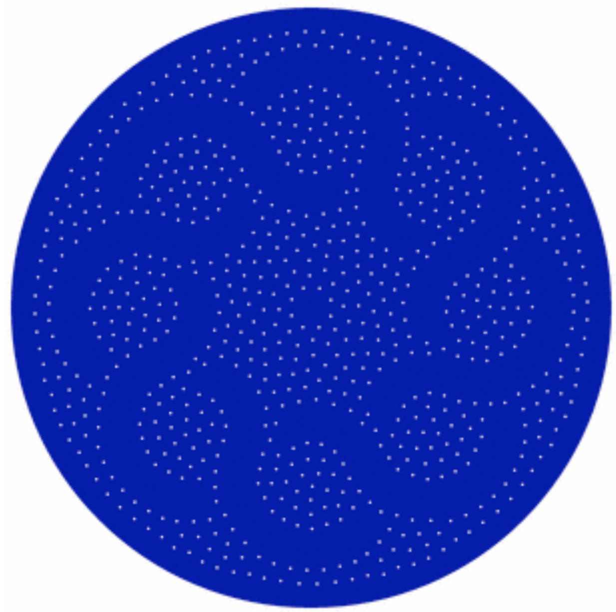
$$\Sigma\ (x_e : \text{all edges } e \text{ in } \chi(l_{A,B})\ ) = 2\ y_{A,B}$$

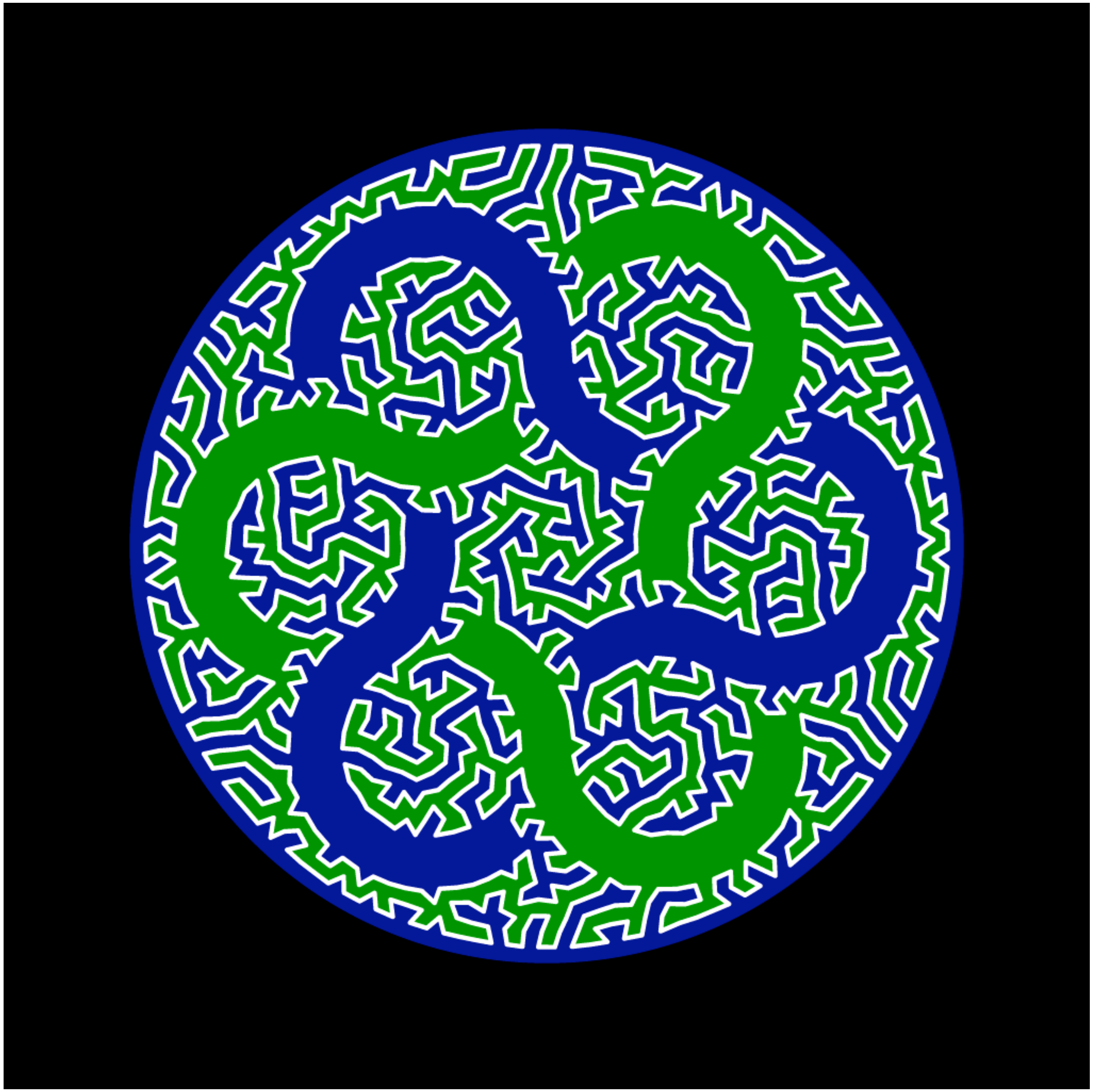
and/or

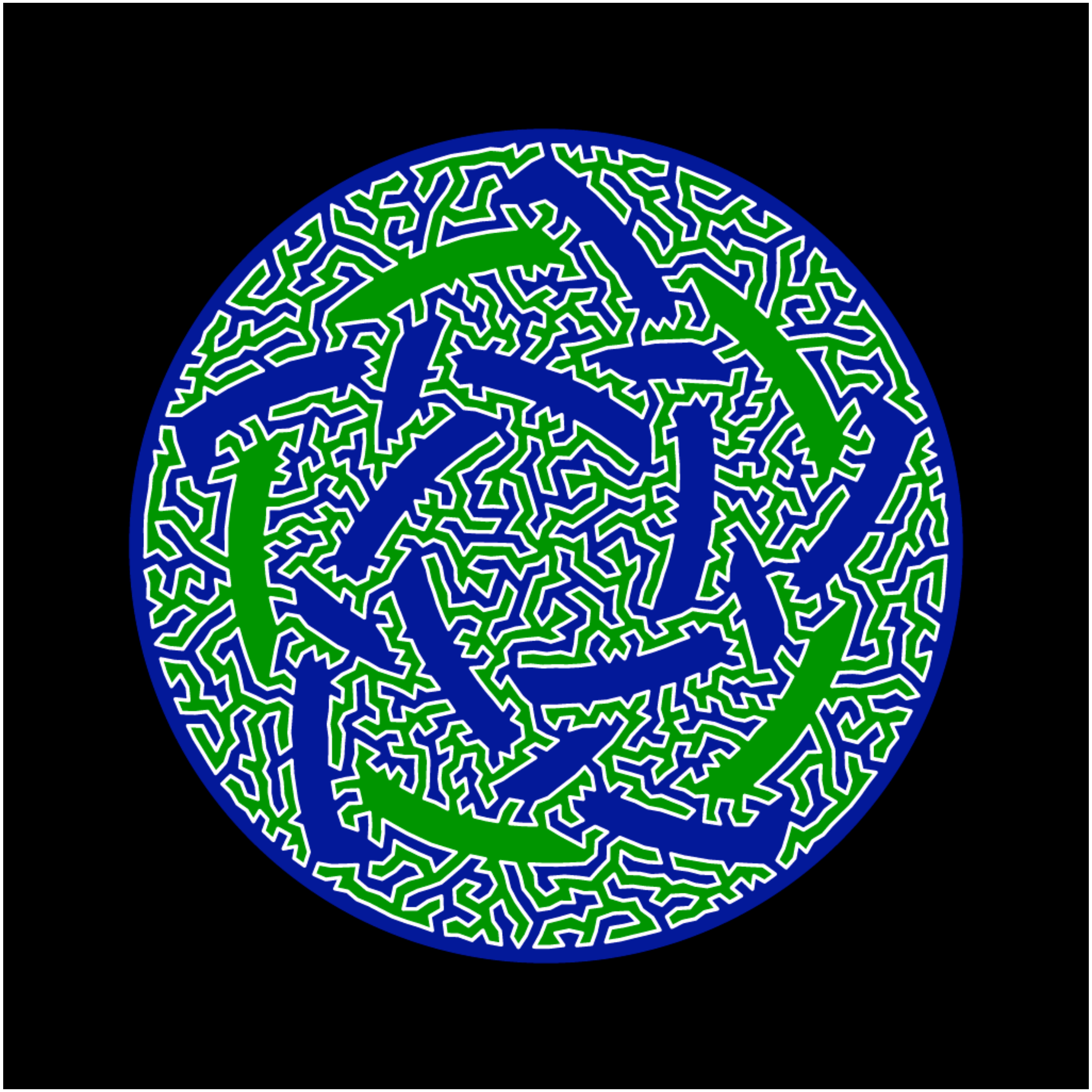
$$\Sigma\ (x_e : \text{all edges } e \text{ in } \chi(l_{A,B})\ ) = 2\ y_{A,B} + 1$$

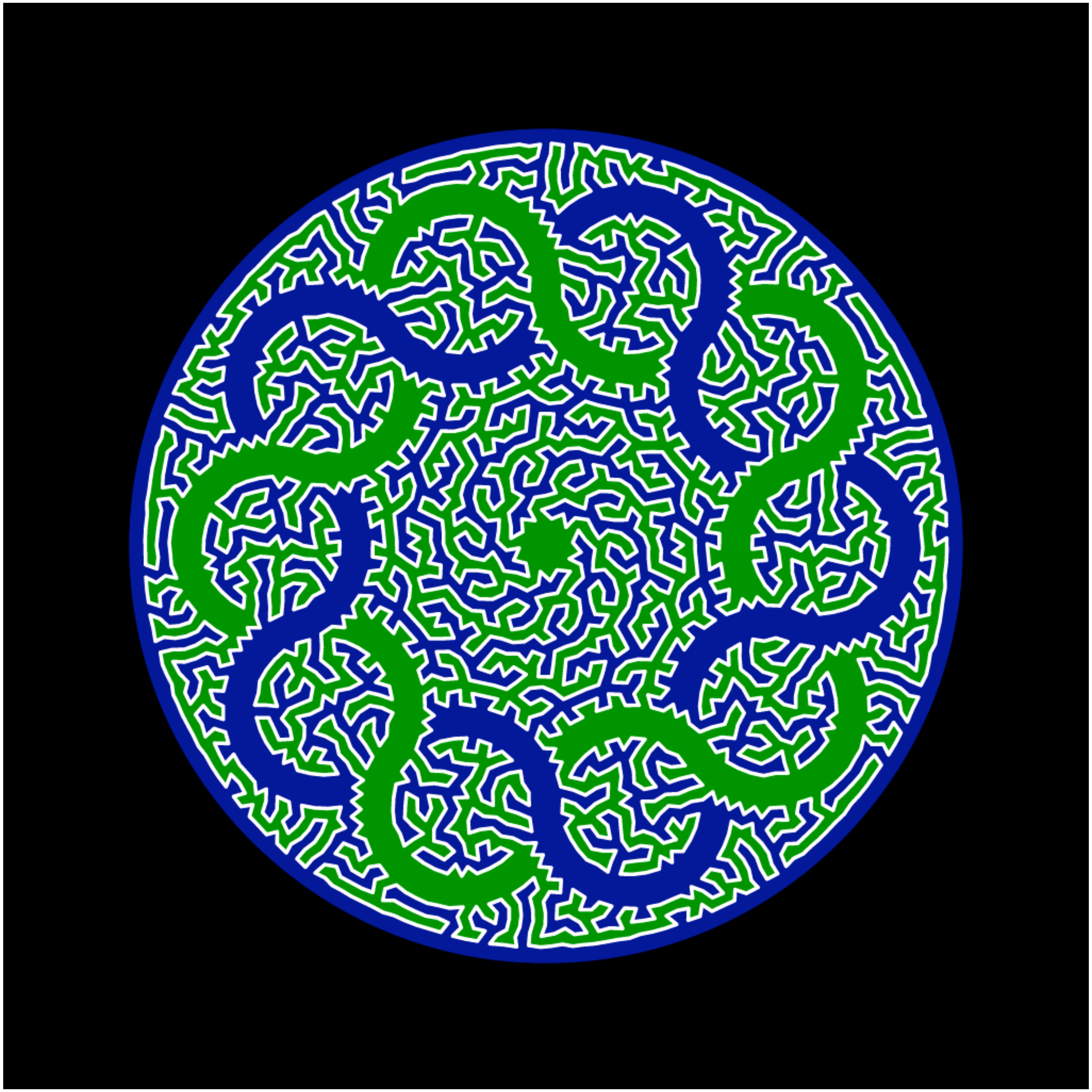
in the DFJ IP formulation.

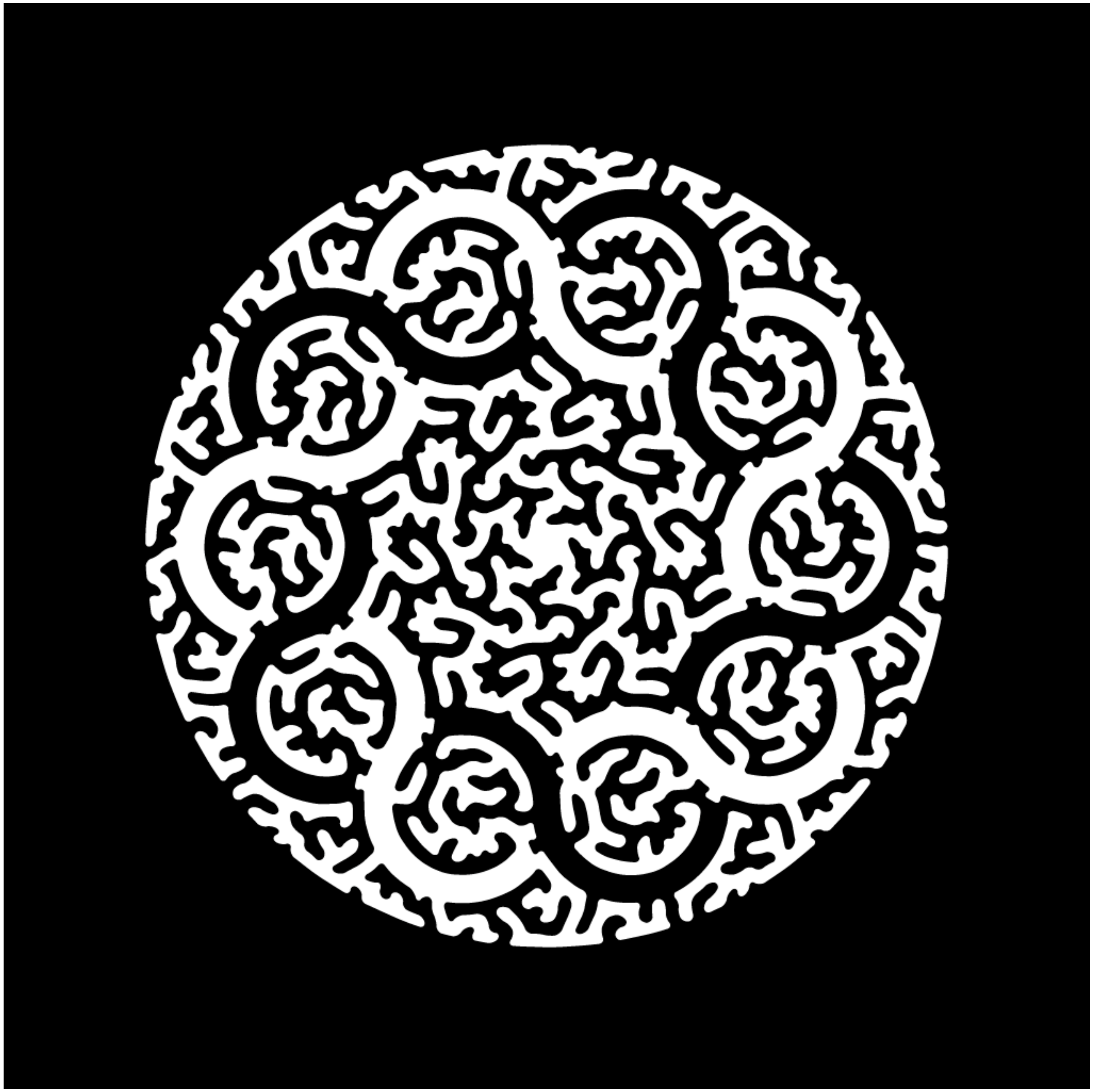


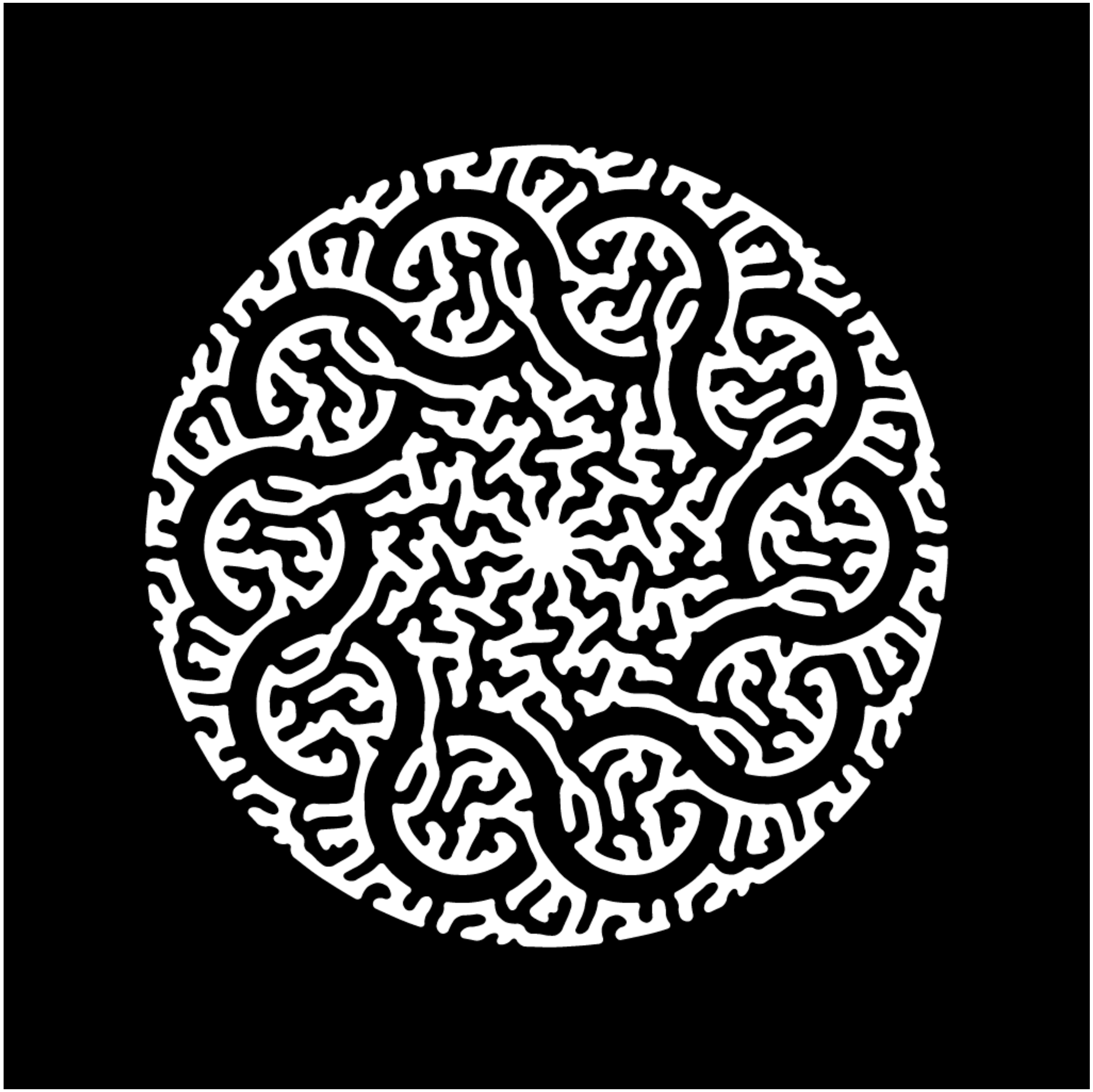


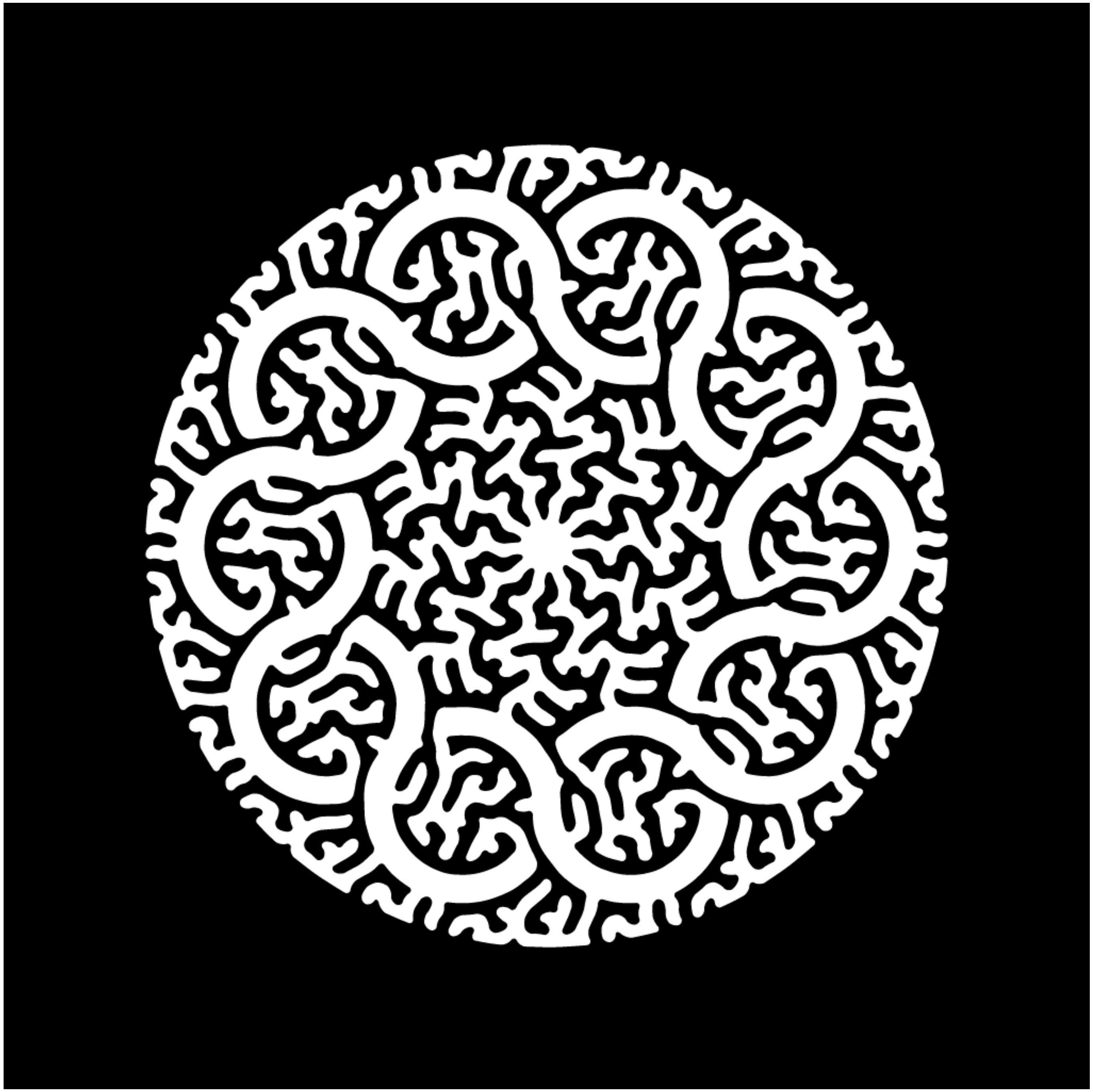












References

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- C.S. Kaplan and R. Bosch. 2005. TSP art. *Bridges 2005.*

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