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Exploring & Visualizing

Hot & Cold Game metaphor

Picture as exploration

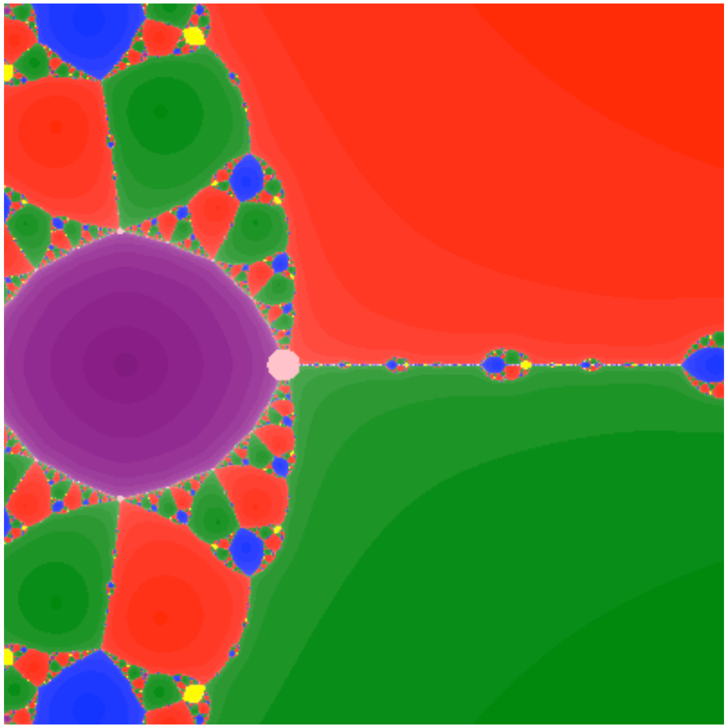
- Most of the polynomiography methods are based on root-finding
- Roots themselves however present little interest
- The important part is the *process* of root-finding rather than *result*

Goal

- Artistic point of view: suggest a new brush
- Mathematically: suggest new algorithms
- Not root-finding algorithms but rather what to do with existing algorithms

The standard algorithm

- Every point in the plane is assigned two numbers: speed of convergence and root index
- The two numbers are later transformed into a color



Speed of convergence

- Pick up a point in the plane
- Iterate it (repeatedly apply a certain procedure)
- Stop based on some criterion
- Record how many times you did it

That’s number of iterations =
speed of convergence

Root index

- Polynomials, or functions in general, typically have more than one root
- The root index is a whole number recording to which root we arrived = our final destination

Coloring Algorithm

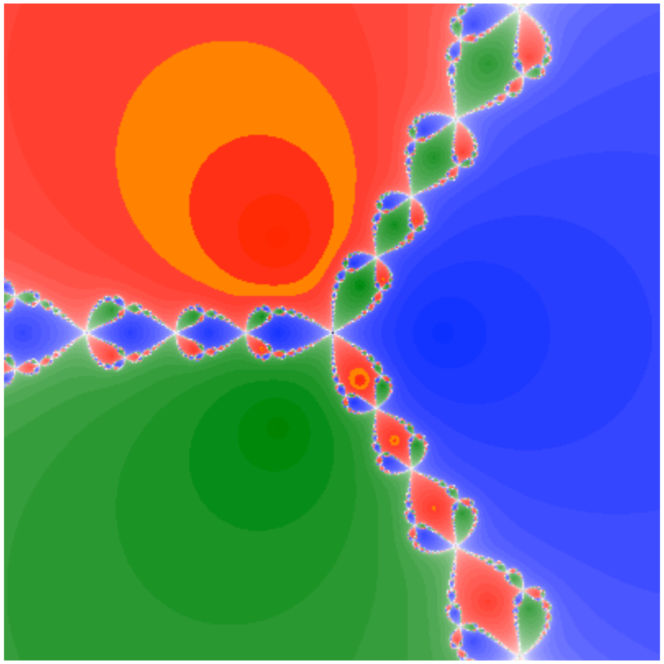
- Typically the root index determines the *color* of the point (red, green, blue, etc)
- The speed of convergence (number of iterations) determines the *hue* of the *color*

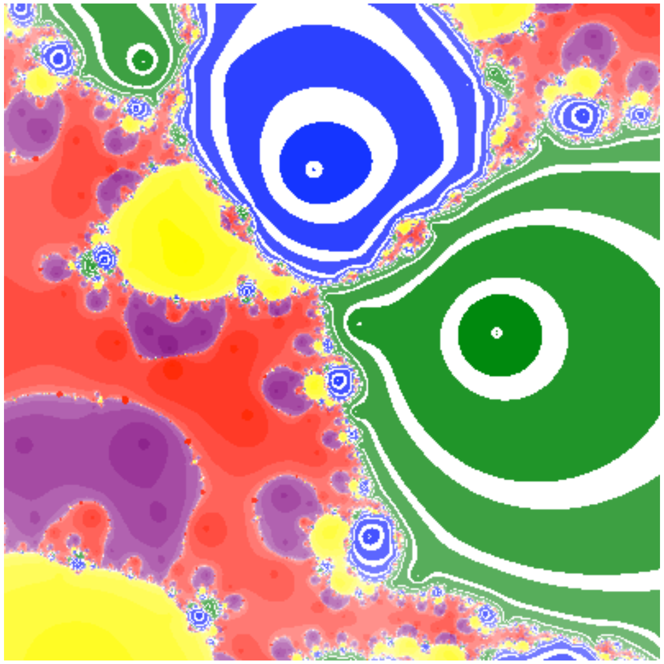
Math is difficult, painting – not

- Difficult part: iterating points and computing the two numbers for every point we want to paint
- Easy part: Reassigning color for the two given numbers

Typical Results

- Smooth parts = continuity areas
- Fractal = difficult = interesting areas





Goals

- Predict the fractal parts without too much computing
- Possibly suggest some root-estimating or root-proximity methods (as opposed to root-finding methods)
- Get cool pictures in the process!
- [end of introduction]

Root = object we seek

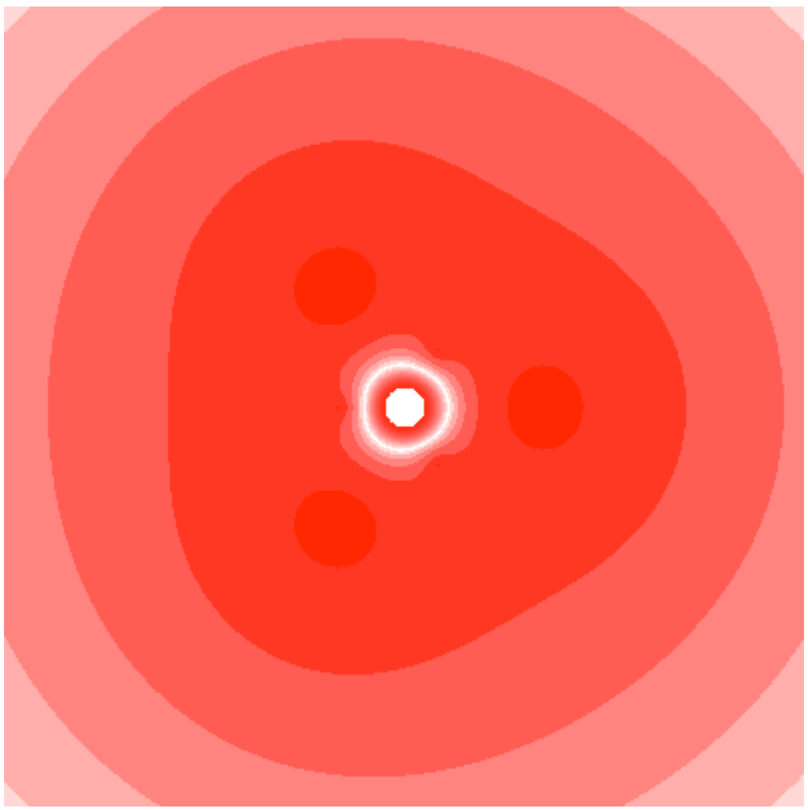
- When we are at the root, we know it
- Even when not at the root we know if we're close or not
- Use |p(x)|, the absolute value (norm) of the polynomial computed at point x
- Norm = absolute value of the polynomial = temperature

Game of Hot and Cold

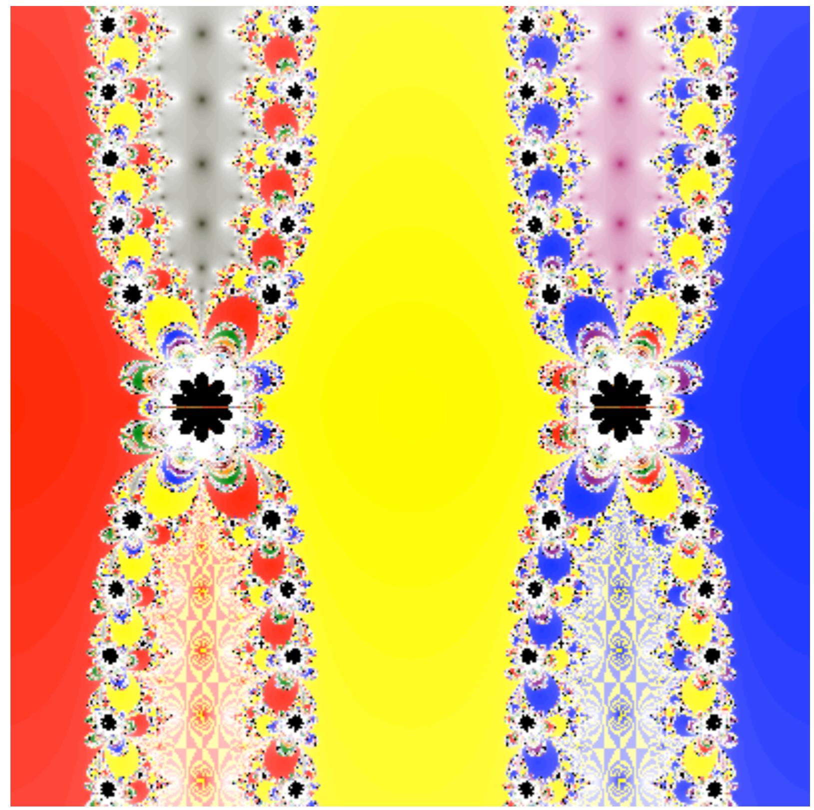
- Hide an object (root)
- Search for it (algorithm)
- Follow hints (numerical data)
- Hot = you are close
- Cold = you are far
- Warmer = you are getting closer
- Colder = you are moving away from it

Bridge between continuous and fractals

- Smooth and continuous



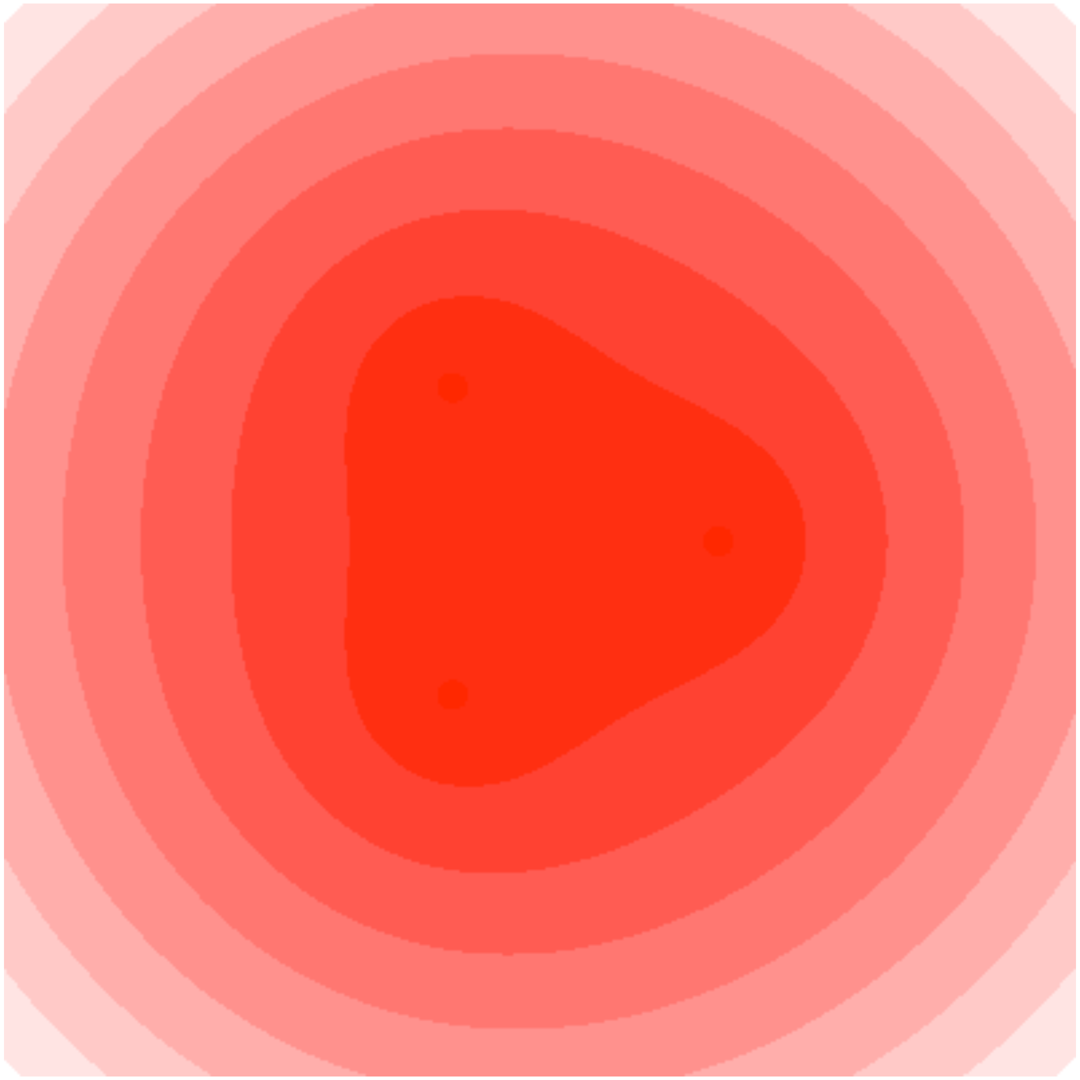
- Fractal and discontinuous



Method 1: Plotting

- Follow the analogy with the temperature
- Plot the "temperature" = absolute value of p(x) over the given two-dimensional area
- Temperature map with red areas around the roots (hot) and white (cold) areas with high values of p.

Method 1: Picture



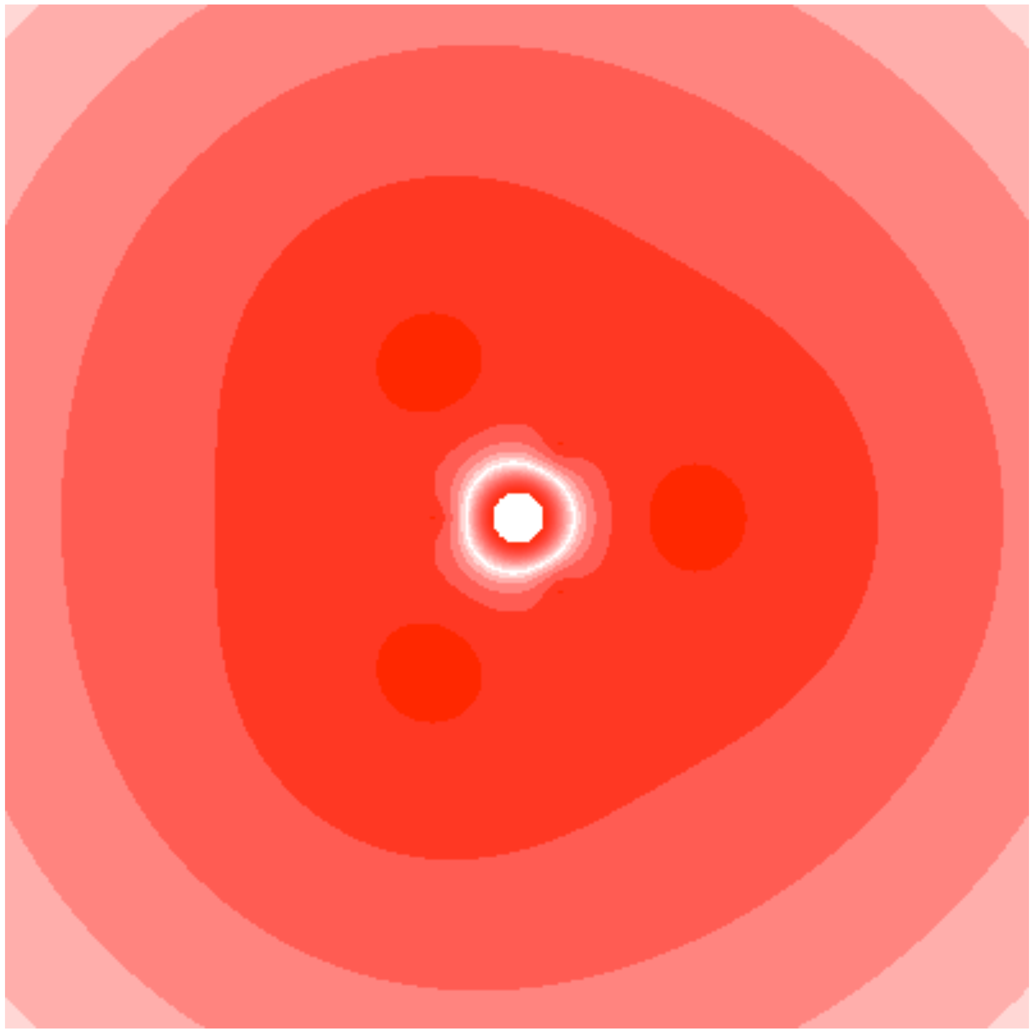
Game-to-Math Dictionary

Game	Math
A (big!) house with objects hidden inside	Polynomial p(x)
Our search strategy	Iteration function
Hidden objects	Roots
Where we are	x
Our next position	
Temperature at where we are	p(x)
… at where we'll be	p(φ(x))

Method 2: Peeking and plotting

- If I enter that room: will it be hot or cold?
- Instead of plotting the temperature at where we are, we plot at where we will be
- Plotting $|p(\varphi(x))|$.
- If it is good a point and good searching strategy, $\varphi(x)$ will be close or closer to a root

Peeking: Picture

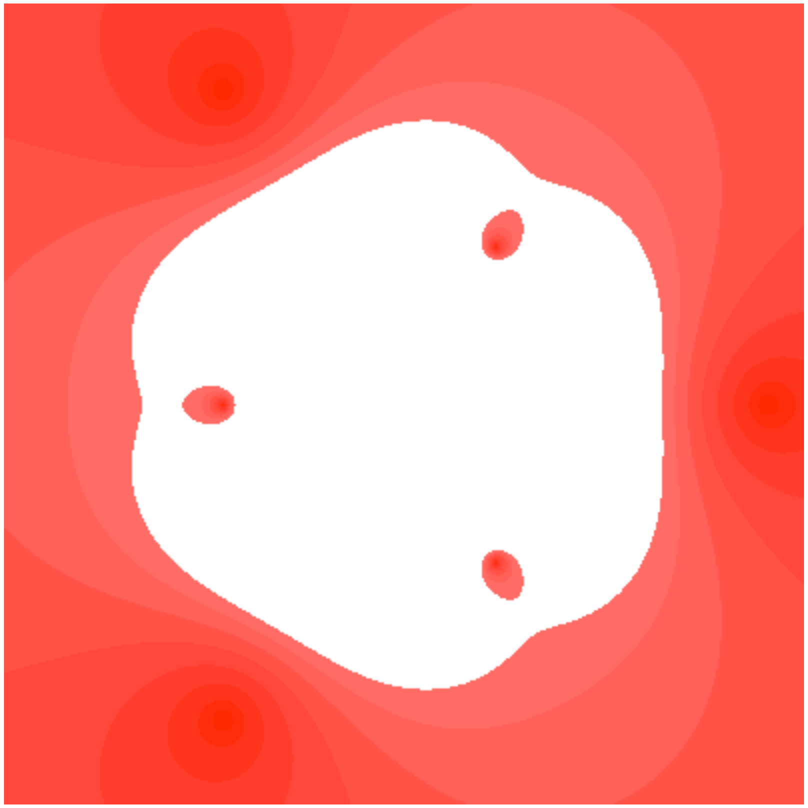


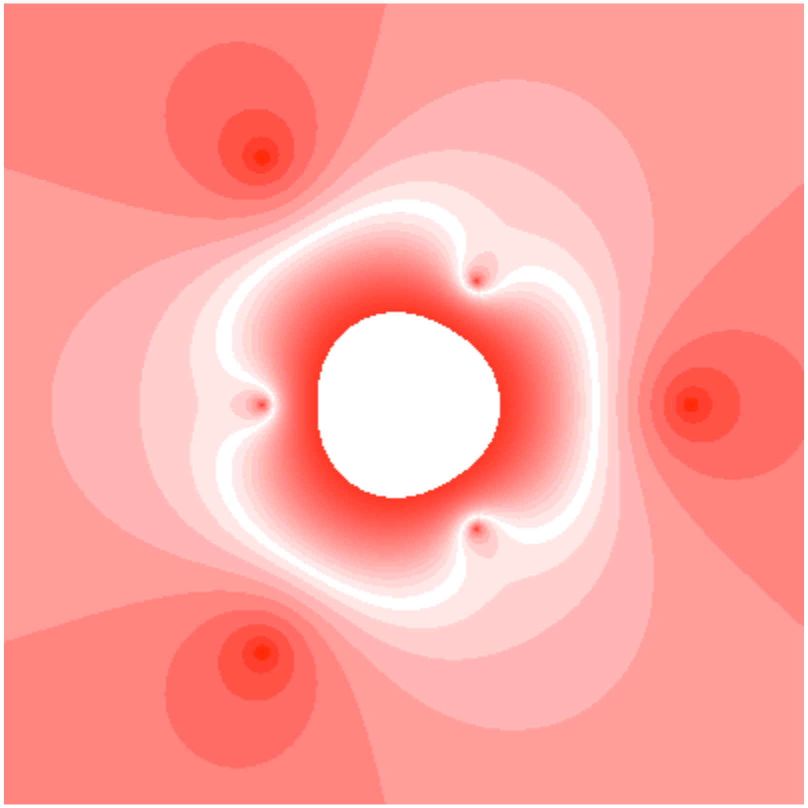
Method 3: Relative

- If I enter that room: will it be hotter or colder?
- Plots relative increase or decrease in temperature
- Plotting

$$\frac{|\,p(\varphi(x))\,|}{|\,p(x)\,|}$$

Relative plotting

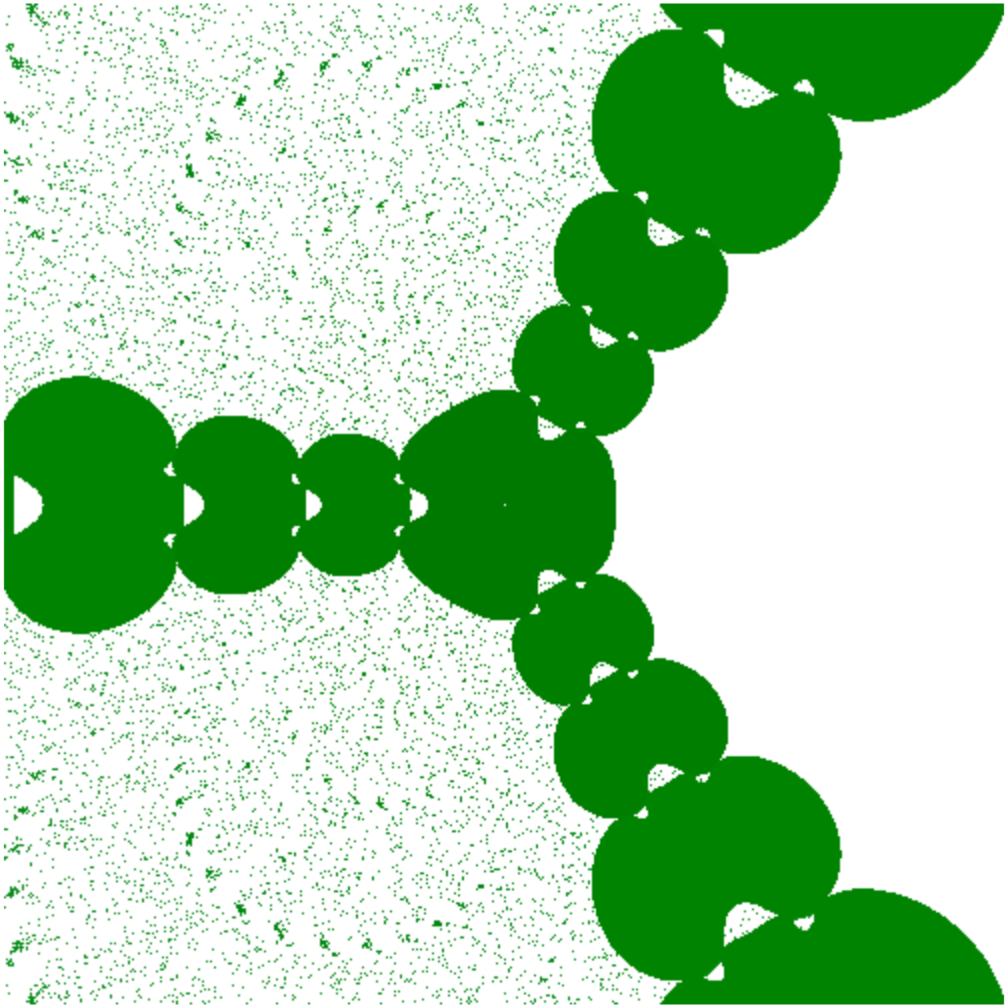


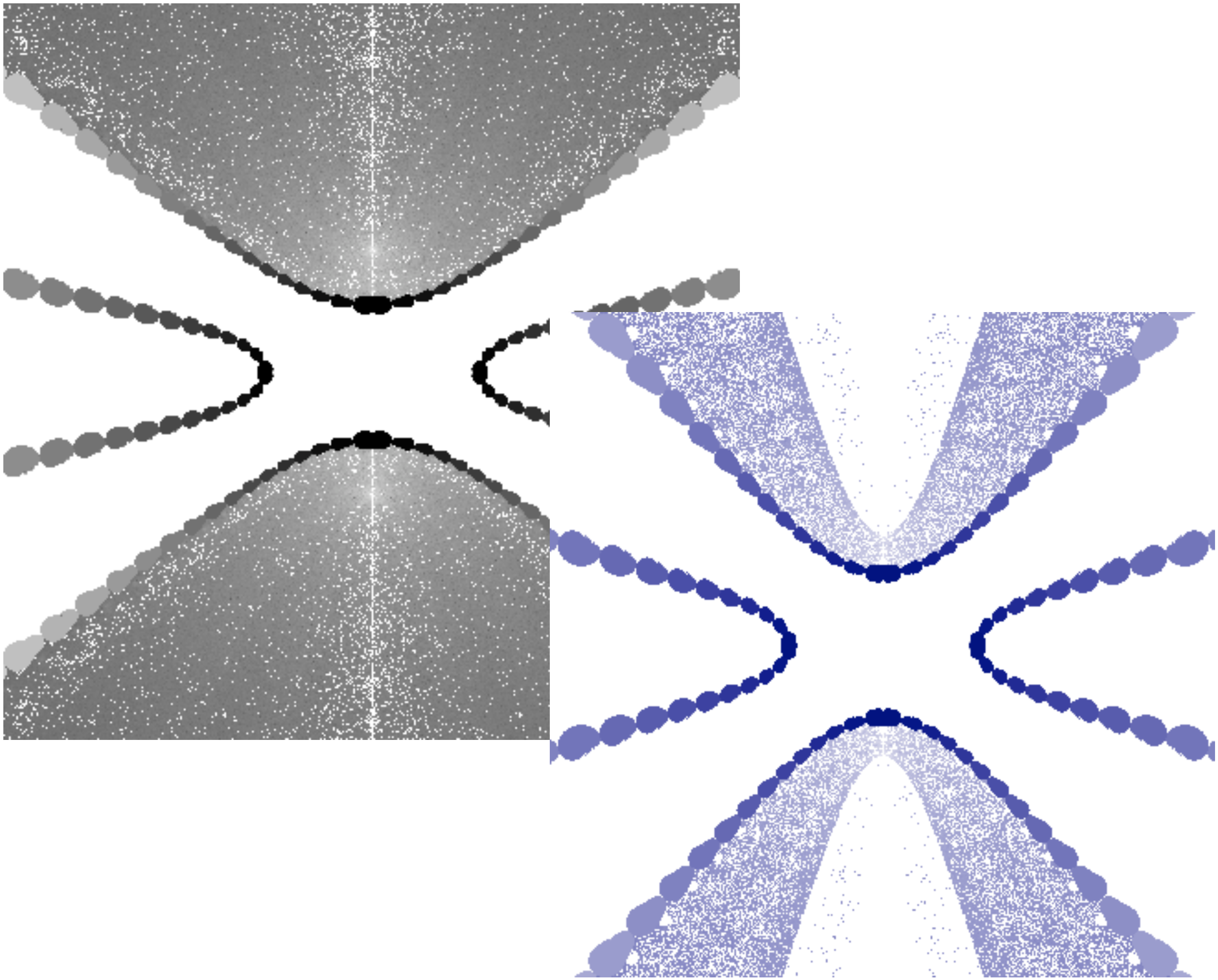


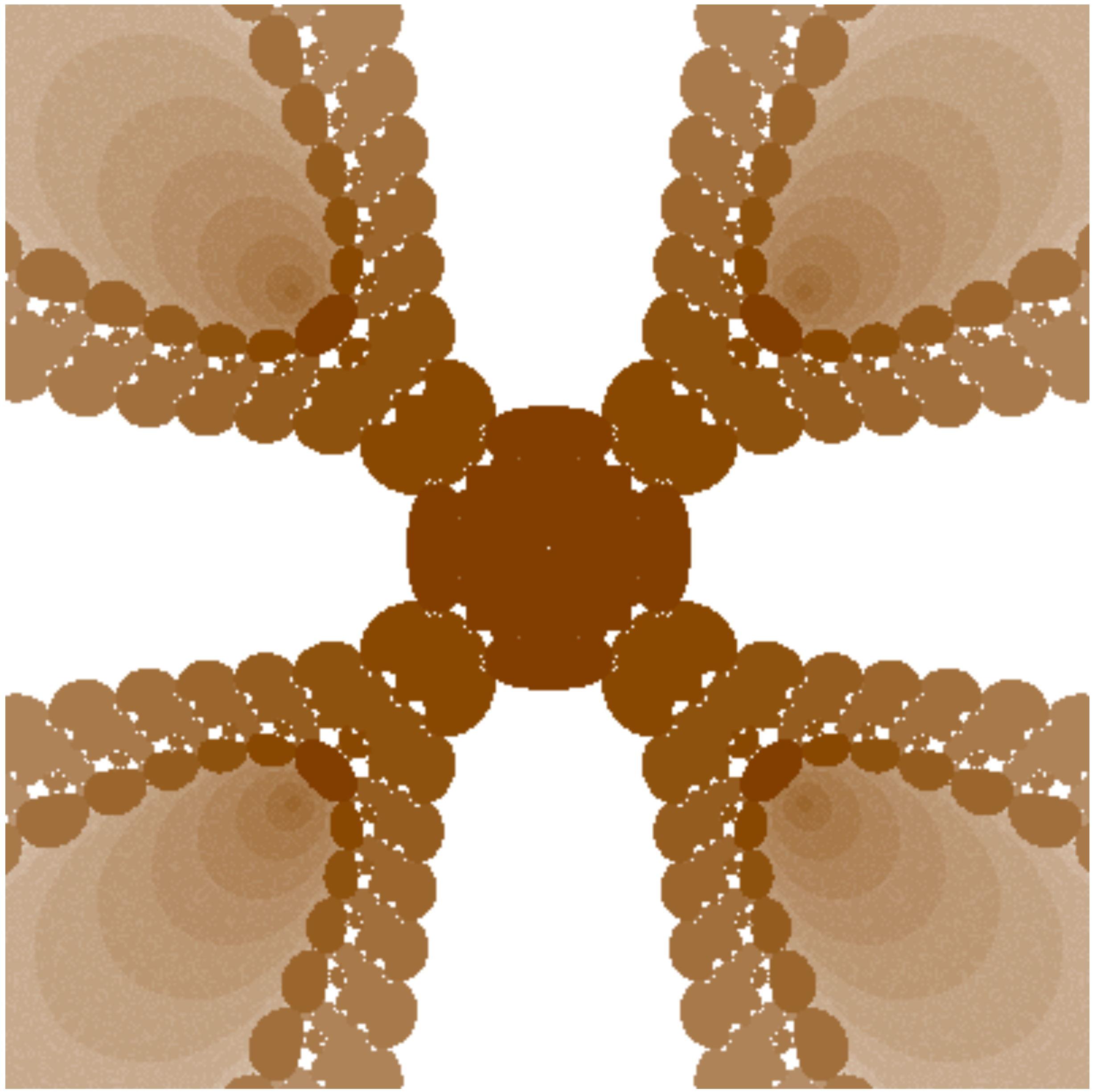
Method 8: Decreasing sequence

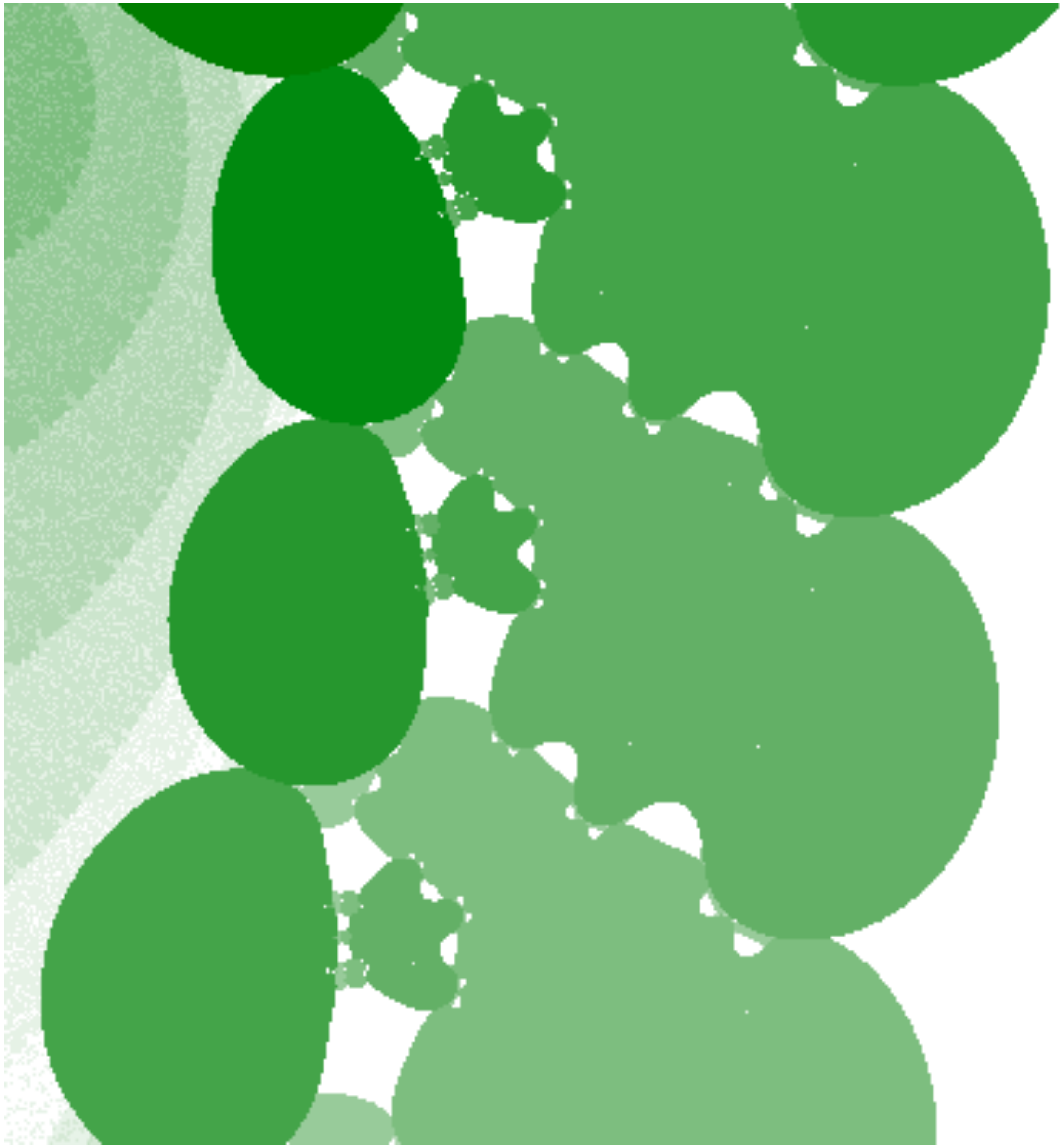
- Traditionally we look at the sequence $x_{i+1} = p(\varphi(x_i))$
- Terminate it when we're close to a root
- In this new method, we terminate the sequence when we're going away from the root (measured by $|p(x)|$)
- Stopping at your destination vs stopping when we hit dead end

Decreasing sequence: Picture









Thank You!